# *Impact Tsunami: Three Ways to Go-*

## Semi-Analytical
## Tsunami by Formula
## Tsunami Squares

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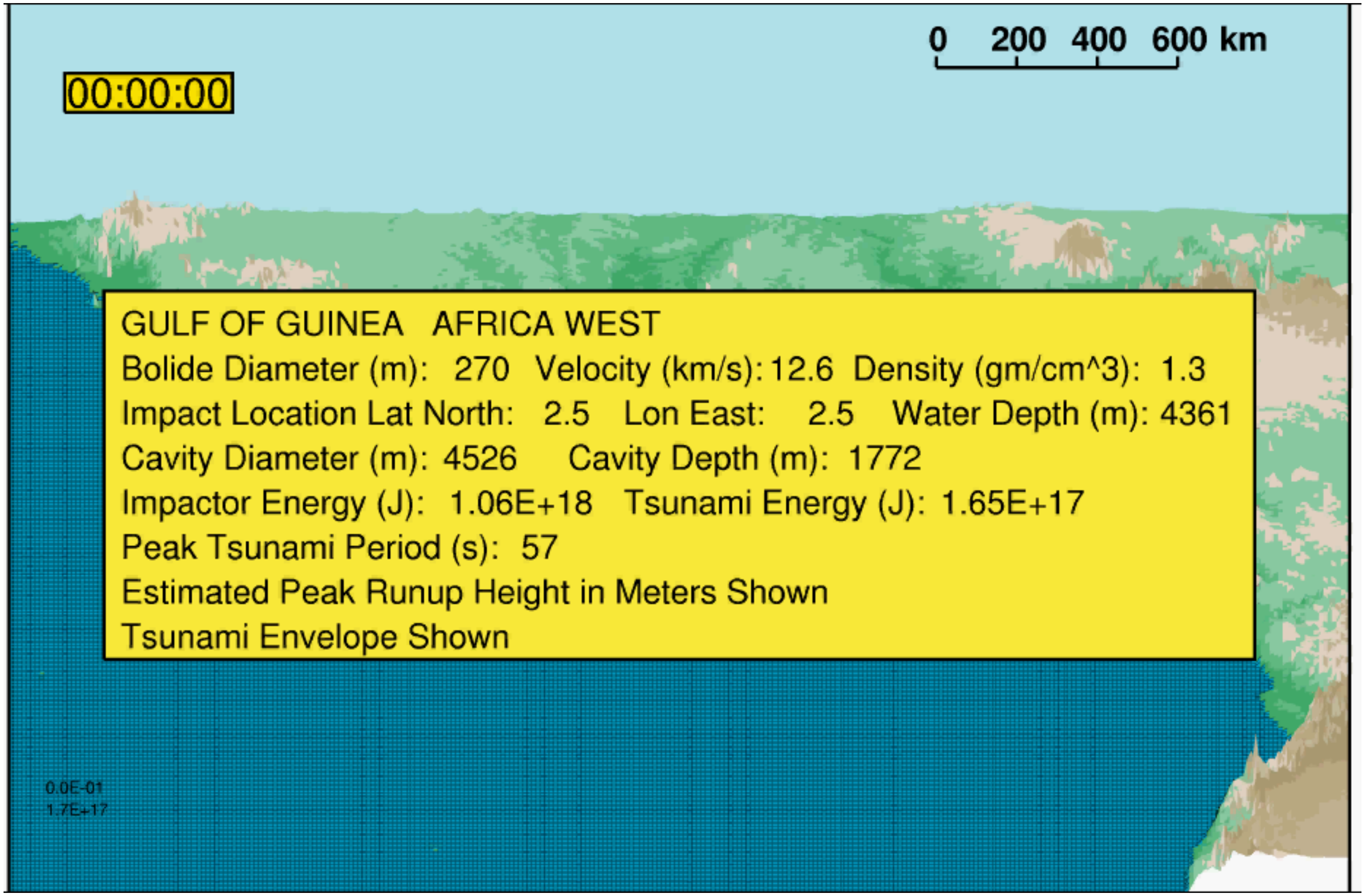
# Semi Analytical



# Semi Analytical

The semi-analytical approach uses linear, dispersive wave theory to derive tsunami waveforms given the initial shape of the transient cavity and/or vertical velocity of the ocean surface.

Under those assumptions, the results are "exact" for uniform depth oceans. A typical expression for vertical water displacement at distance **r** and time t would be

$$u_z^{surf}(\mathbf{r},t) = \int_0^\infty \frac{k\,dk}{2\pi} \cos[\omega(k)t] J_0(kr) F_0(k)$$

Where

$$F_0(k) = \int_{\mathbf{r}_0} r\, d\mathbf{r}_0\, u_z^{impact}(\mathbf{r}_0)\, J_0(kr_0) = \frac{2\pi D_C R_D}{k} J_3(kR_D)$$

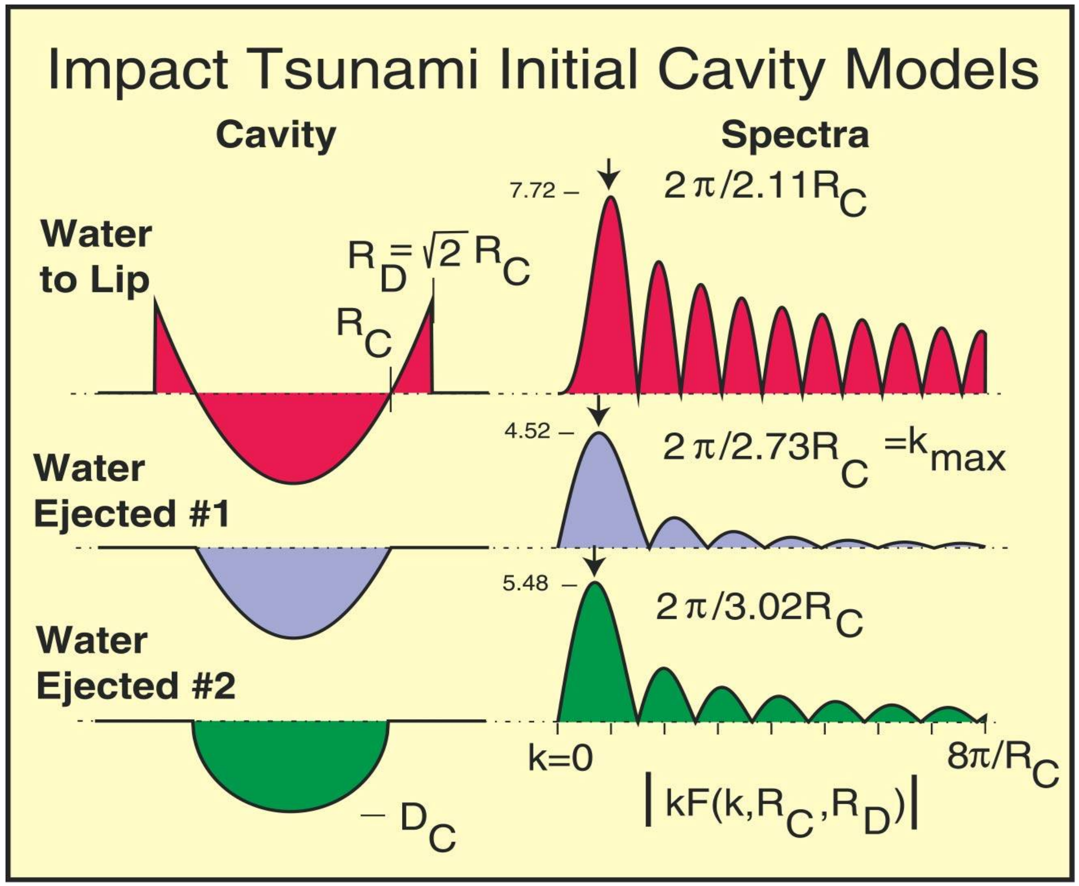
$$u_z^{surf}(\mathbf{r},t) = \int_0^\infty \frac{k\,dk}{2\pi}\cos[\omega(k)t]J_0(kr)F_0(k)$$ The first bit contains all the dispersive and geometrical spreading information.

The second bit $$F_0(k) = \int_{\mathbf{r}_0} r\,d\mathbf{r}_0\, u_z^{impact}(\mathbf{r}_0)\, J_0(kr_0) = \frac{2\pi D_C R_D}{k} J_3(kR_D)$$

contains the information about the transient cavity.

The most important features as far as tsunami are concerned are cavity depth and cavity diameter.

Other details are interesting but secondary.



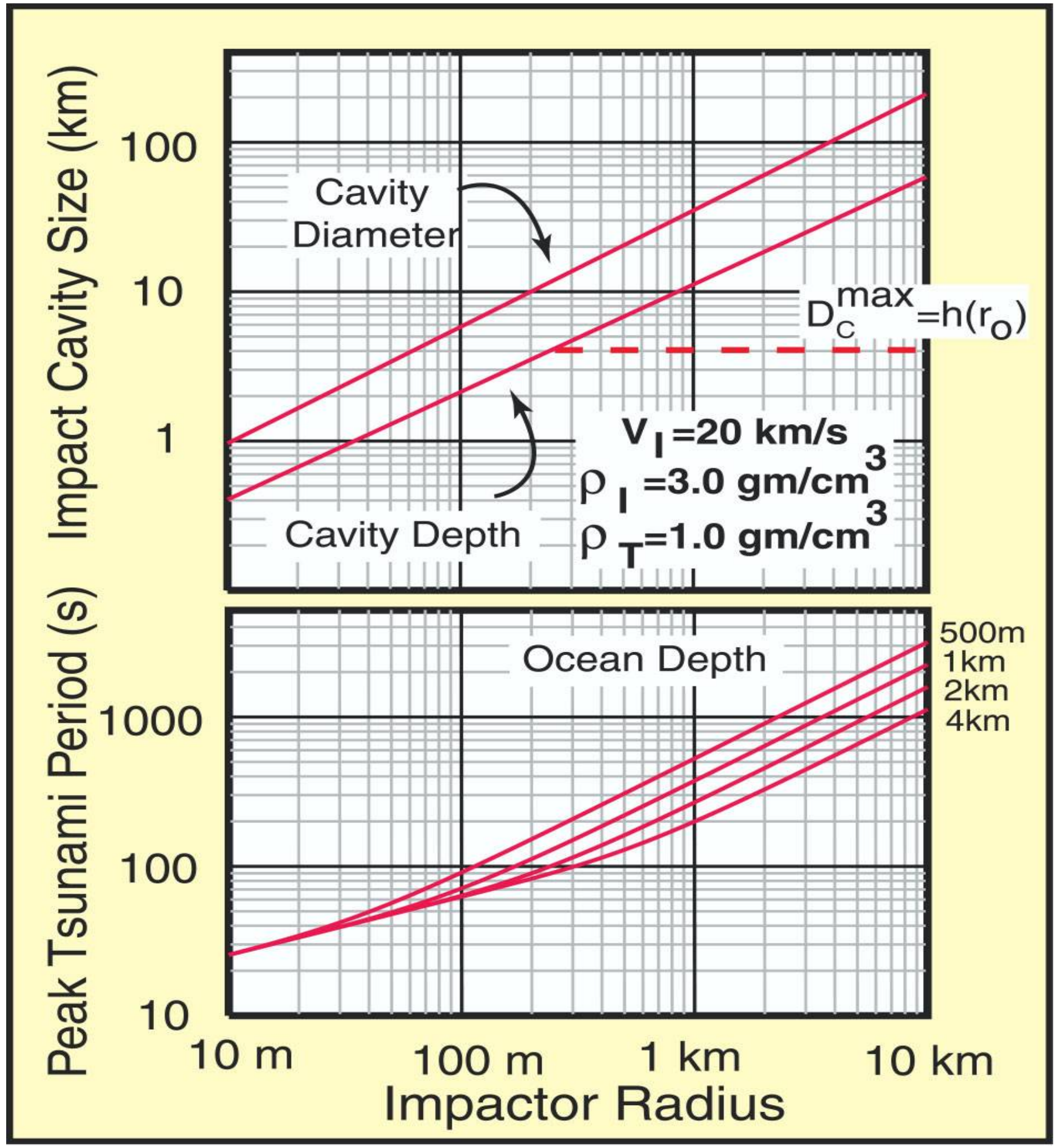

*Bigger cavities produce longer waves. Peak tsunami heights are in waves of length 2 to 3 times crater radius.*

Cavity information versus impactor size comes from scaling laws

Typical cavity diameters are 10-50 km.

Typical tsunami periods from cavities of that size are 70 -150 s

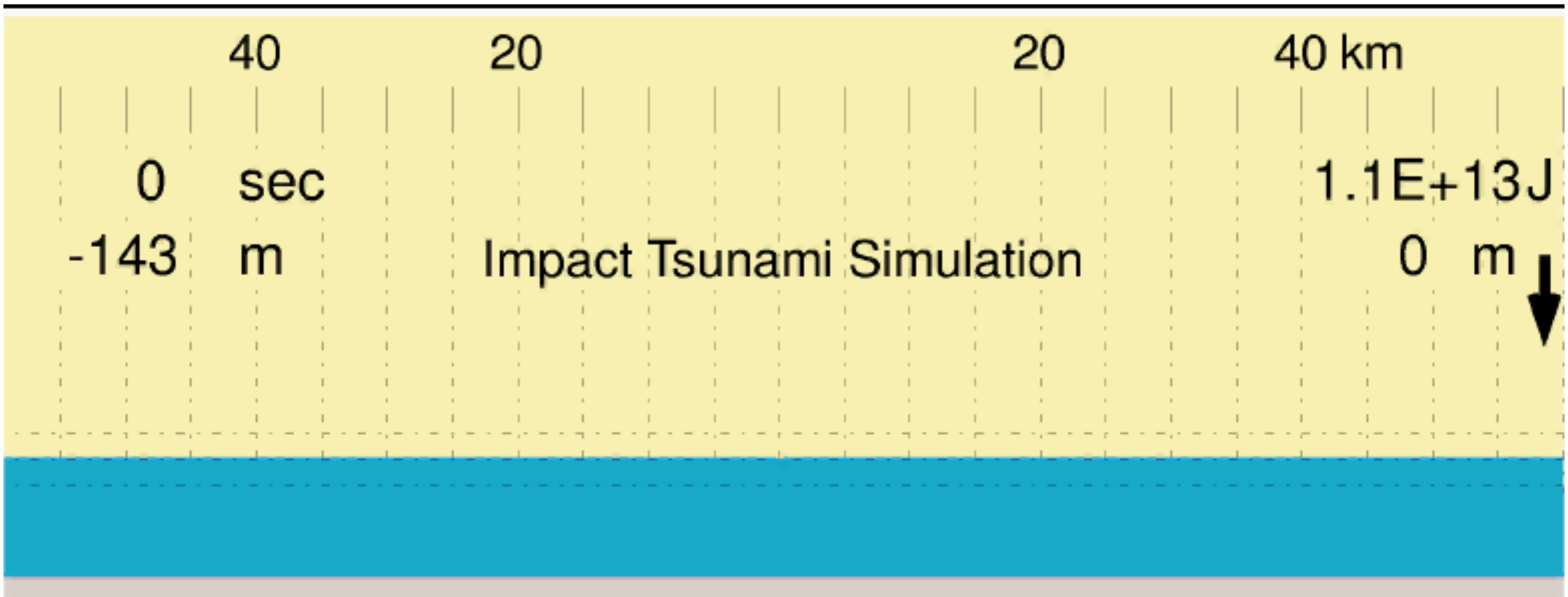
Other than landslides, nothing makes waves of this period, so it is hard to find natural analogs of impact tsunami.



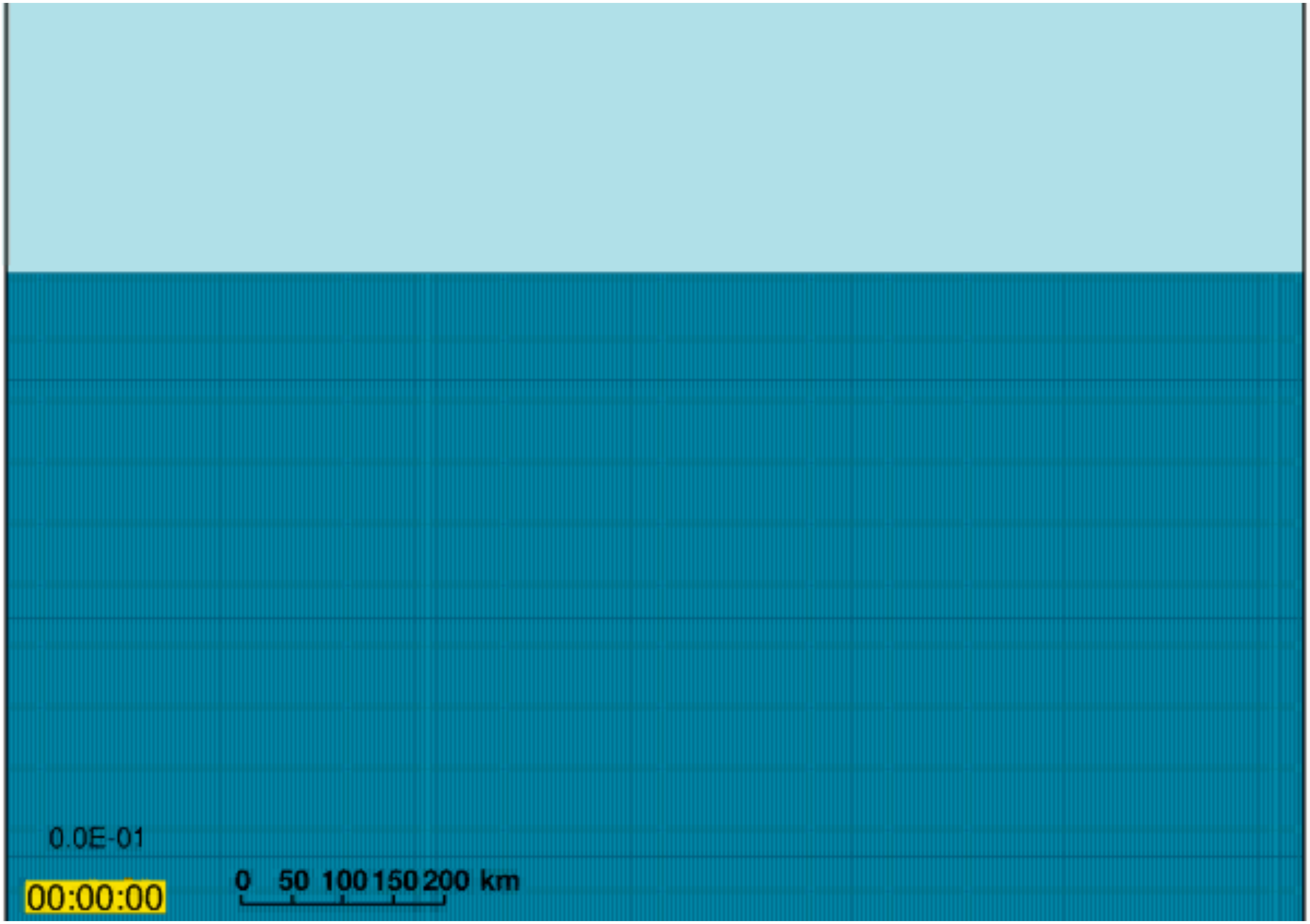

*Cavity Depth limited to ocean depth of course.*

True, linear theory tells us nothing about non-linear, turbulent dissipation. Hydrocodes help, but disagreements exist there too.

Still, as long as the theory predicts tsunami REASONABLY WELL AT SOME DISTANCE AWAY, that’s OK given all the other uncertainties




Impact of 1km diameter asteroid into 4600 m ocean. Cavity rebounds and tsunami waves are sent out. Dark line is hydrocode result by Valery Shuvalov. Simple model doesn’t look half bad to me.

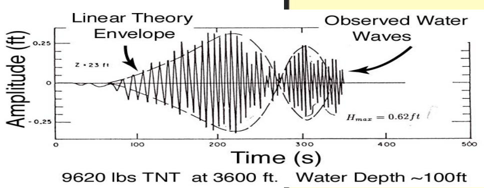



*Impact tsunami in 3d. Uniform Ocean  See the many waves.*

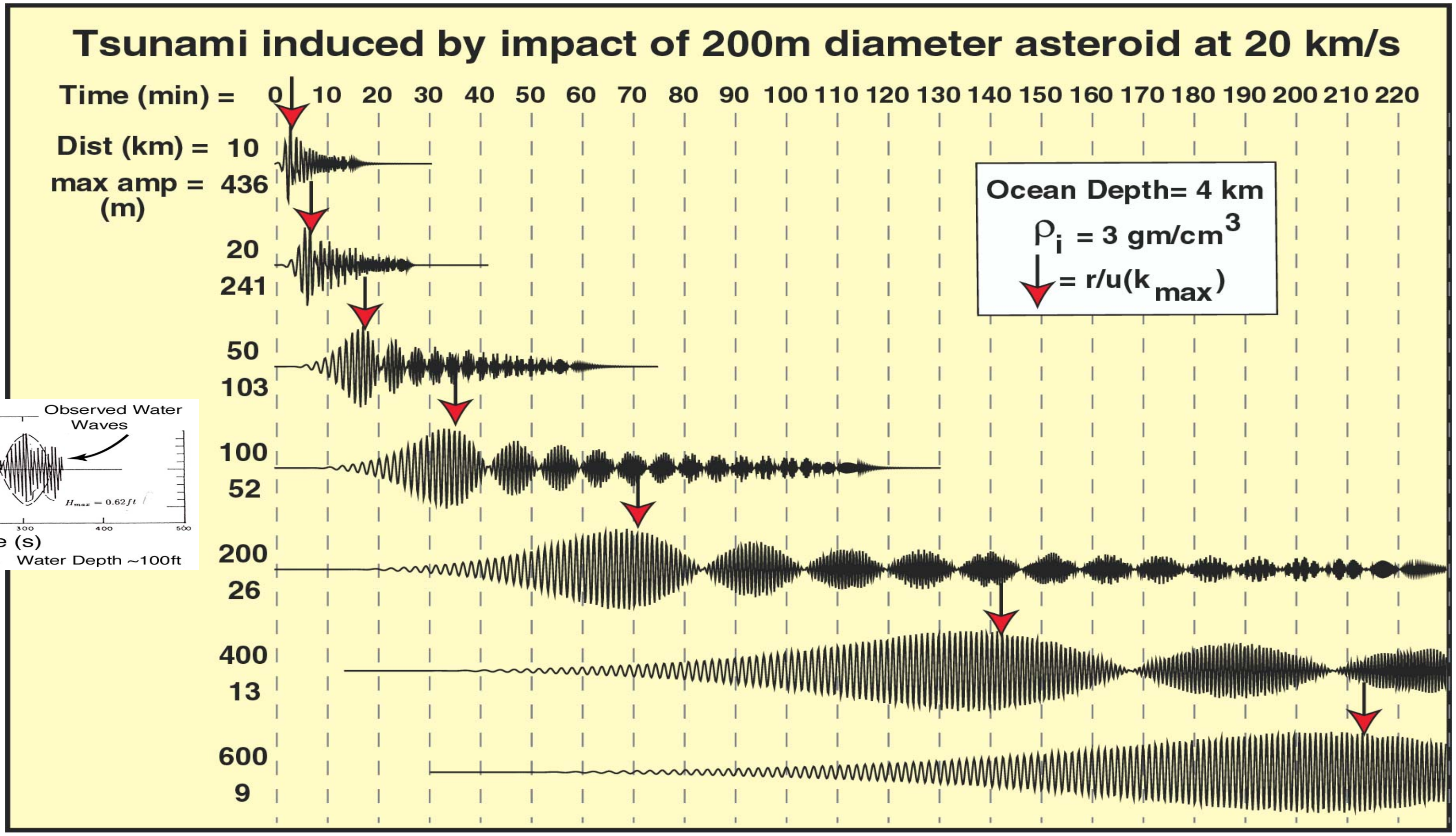
# *Impact tsunami waves versus time and distance*

*(Below.)*

*Real explosion again. Looks about the same*




*as simple model (Right) to me*



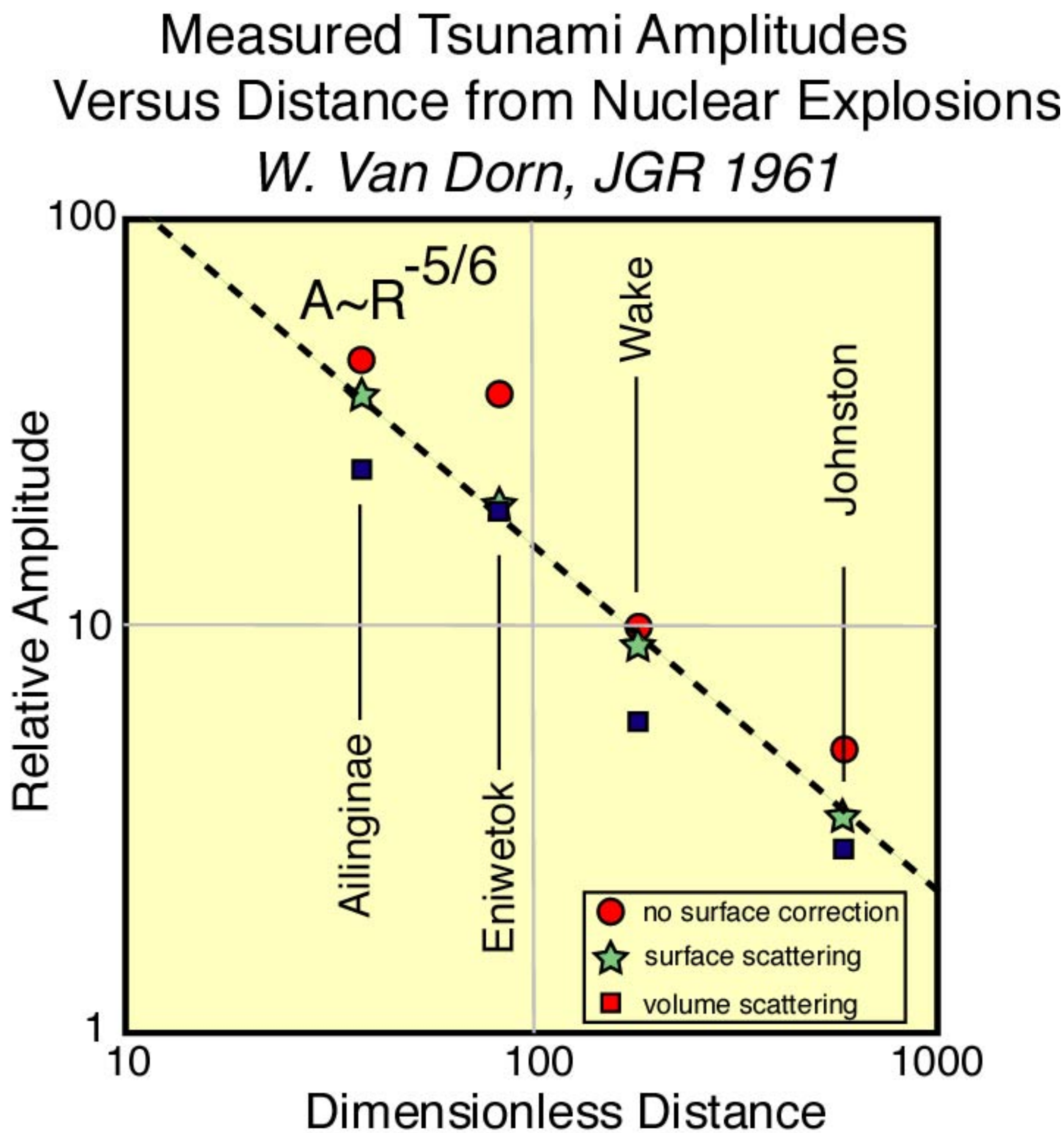

*Impact tsunami are very dispersive. Long periods travel faster than short periods. Dispersion reduces impact tsunami size with distance faster than EQ tsunami.*

*Tsunami are cylindrical waves not spherical waves* ***Geometrical Spreading*** *for tsunami follow $R^{-1/2}$ not $R^{-1}$*

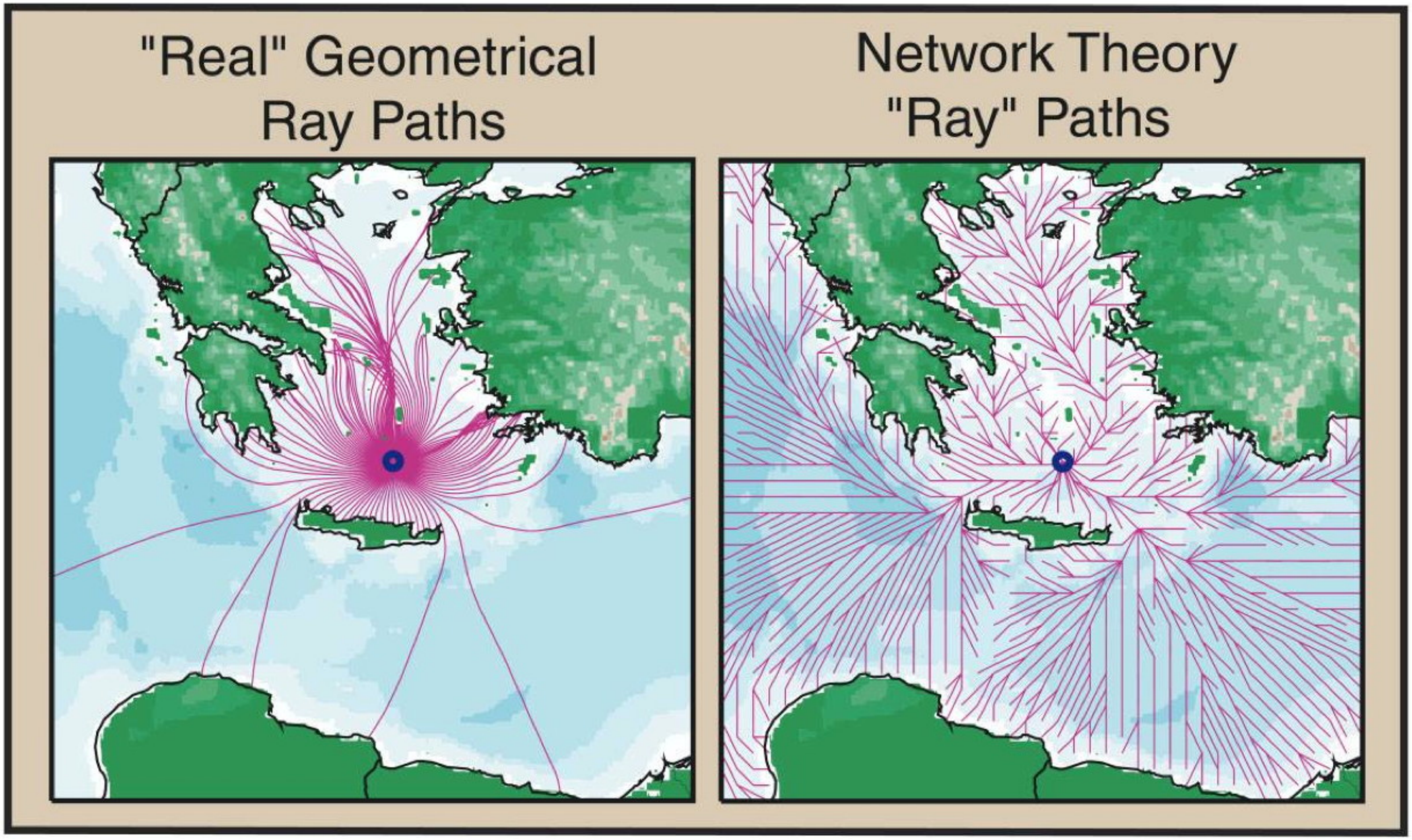
*Frequency Dispersion reduces heights by an additional $R^{-1/4}$ to $R^{-1/2}$*

*In 1961 W. Van Dorn measured tsunami waves generated by nuclear tests.*

*He found $R^{-5/6}$ decay much in accord with linear theory.*



Transition to variable depth oceans by introducing ‘rays’ – either
Real Geometrical Ones or Ones fixed by Network Theory.

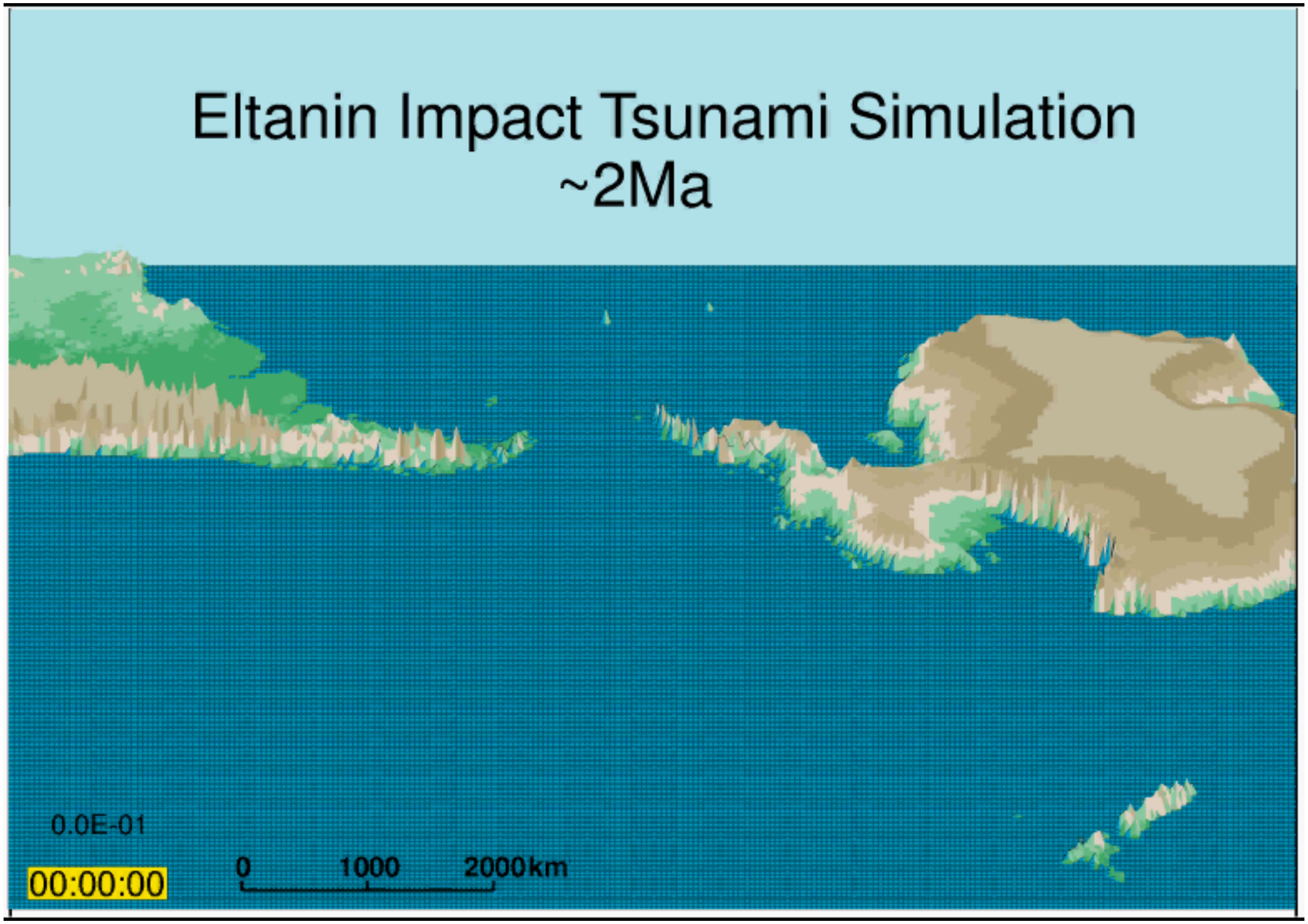


# *Real Example-* *Asteroid Eltanin Diameter: 1100m*

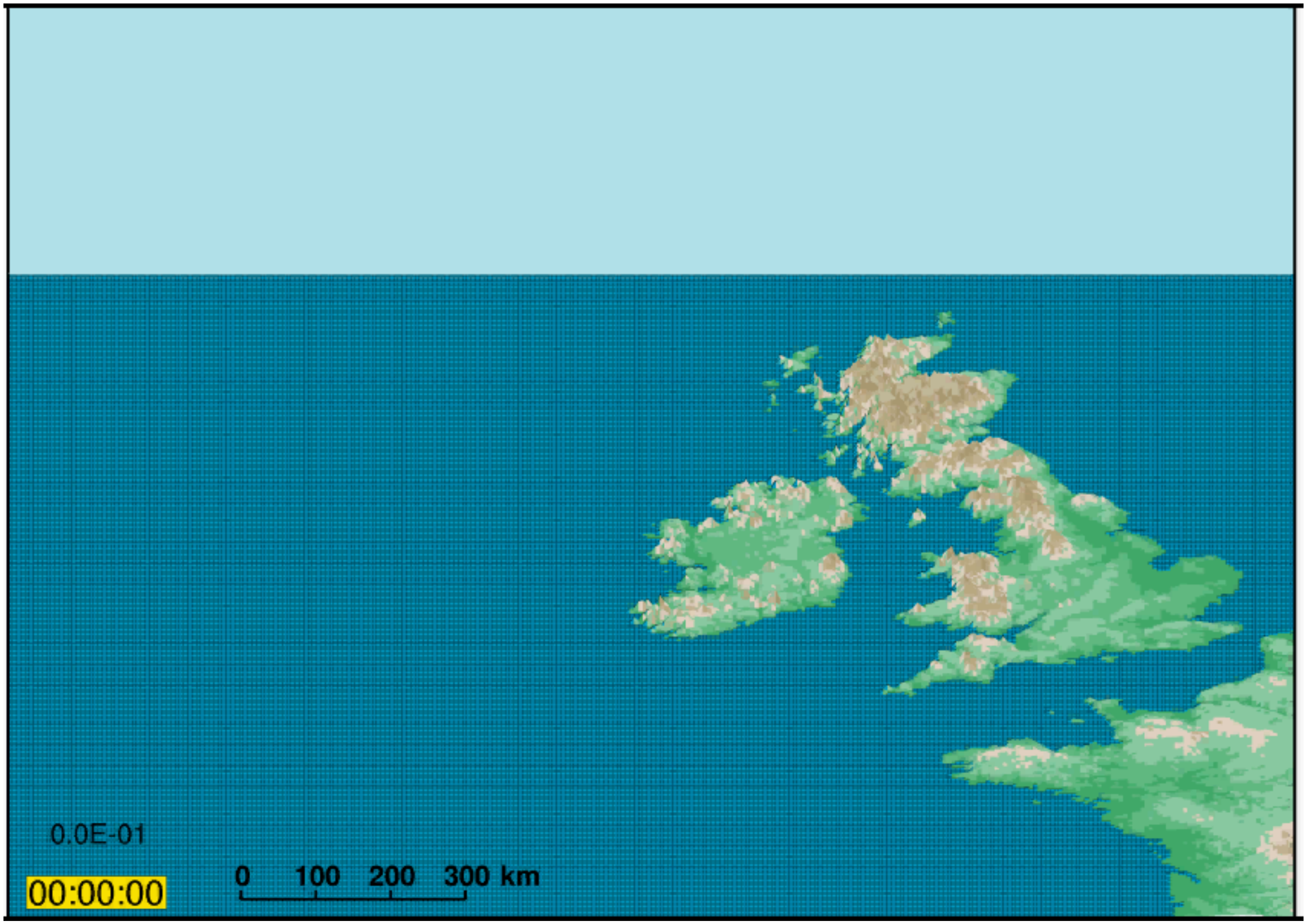
*When: 2.15 Million Years Ago. Where: South Pacific*

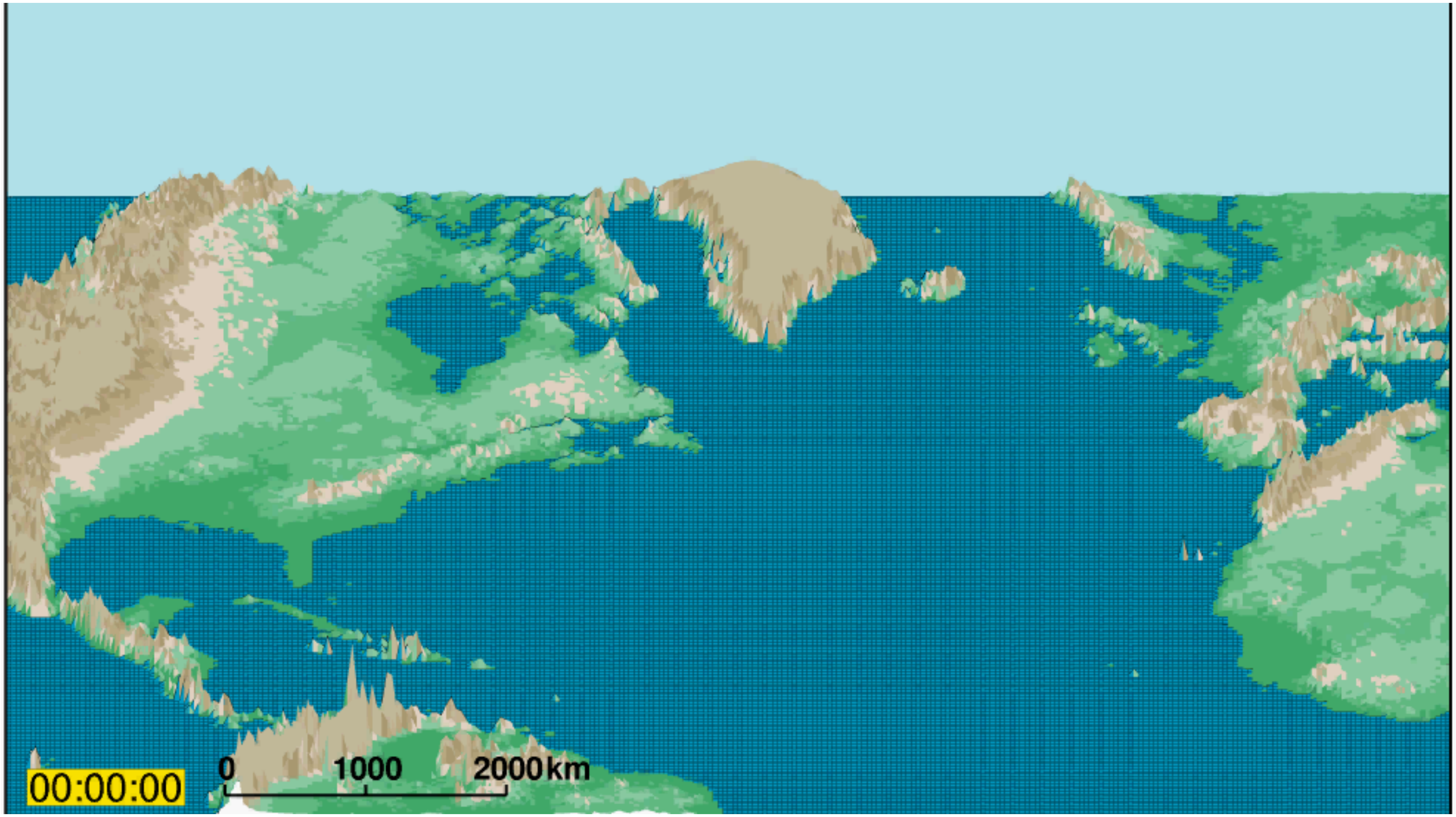
*Waves were ~50 m high in South America.*

*Tsunami this large affect entire ocean basins. Tsunami envelope shown here - many wave cycles underneath the cover.*



*Propagation of linear tsunami well beyond the cavity is not widely in dispute. Tsunami waves crush together in the shallows and "bend around" obstacles. Watch out Ireland.*






*Tsunami take ~8-15 hours to cross ocean basins. Tsunami envelope plotted here again. Runups in meters shown.*

*Yikes, this is a big one*

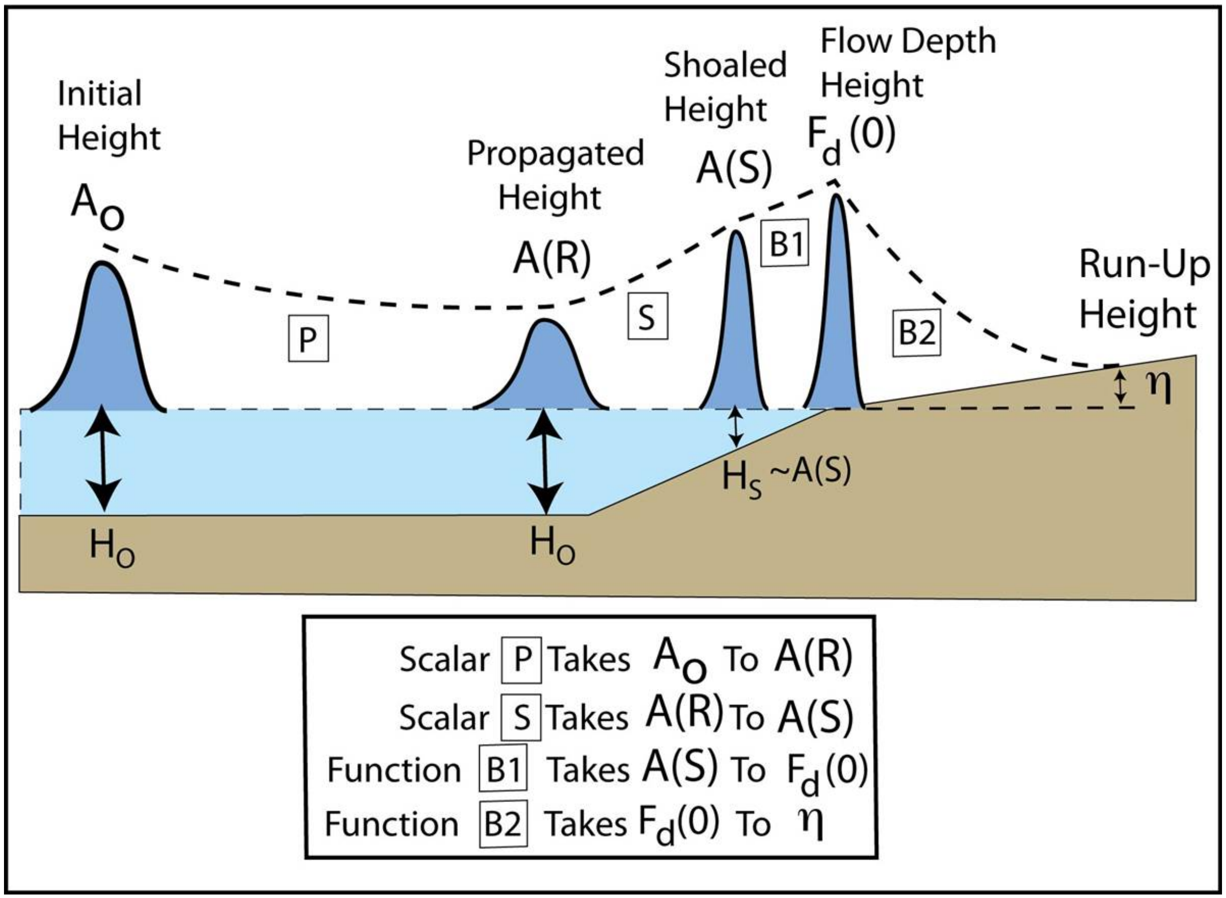
# Semi-Analytical

*Pros:* Calculation is fairly quick. No need to carry the waves through all spatial points from source to receiver. Dispersion is fully included. Little concern about numerical noise or numerical attenuation. Products are depth dependent. No equations to solve. Results are easy to interpret physically.

*Cons:* Results for variable depth oceans only approximate. No account is taken for wave reflections or multi-paths. Purely linear result. Can't carry waves to very shallow water or onto land. Must provide fairly simple initial conditions.

# Tsunami By Formula

## *Run Up Flow Chart*



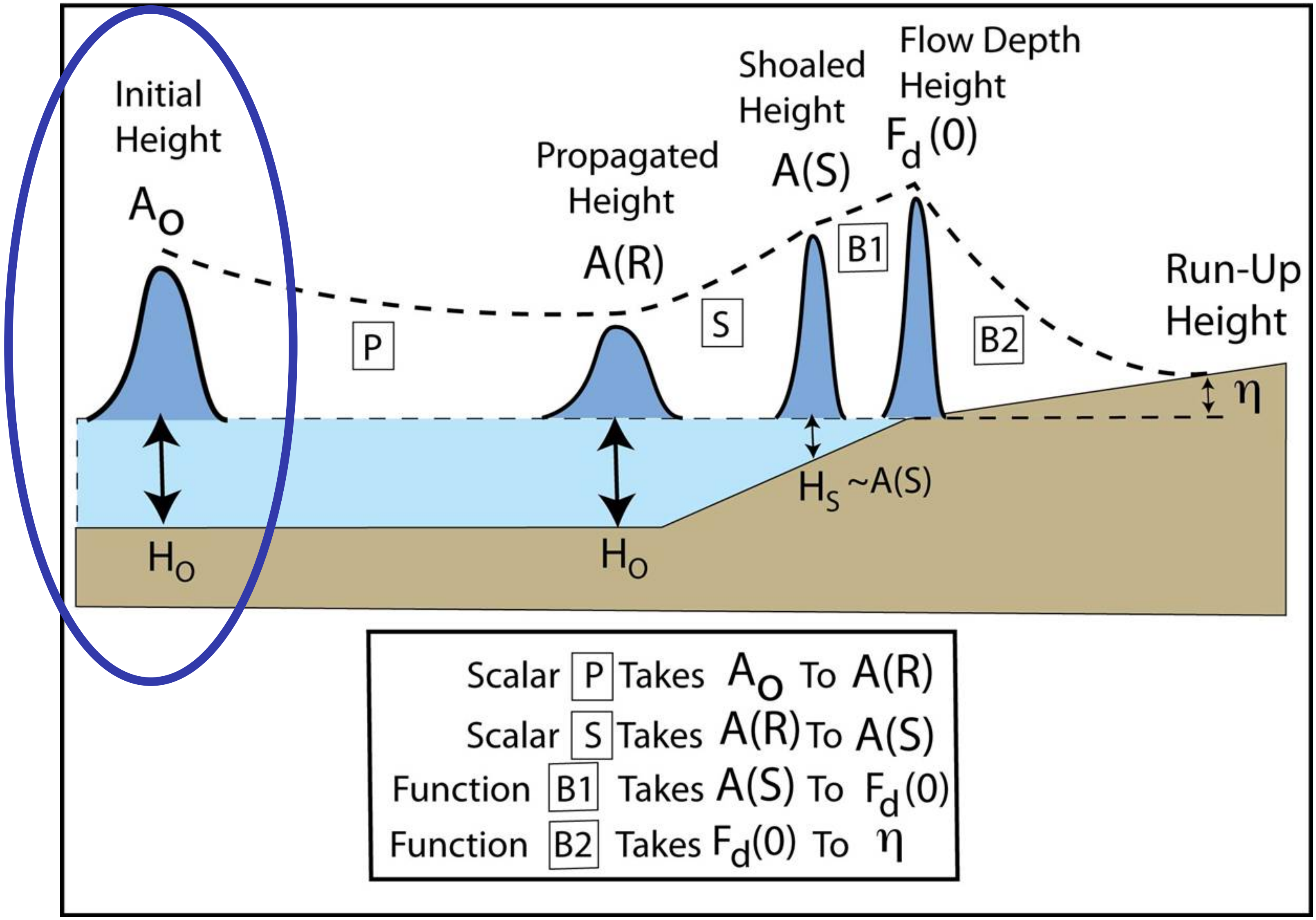
# Tsunami by Formula

There is a demand for quick and dirty estimates of peak wave run up given very limited knowledge of the tsunami source or intervening geography/bathymetry. I call this approach "tsunami by formula".

In its barest form, tsunami involve just a few stages. By making many semi-analytical runs of various sources, in various water depths, at various distances. It is possible to reduce each of the tsunami stages to a scalar multiplication or elementary functional output.

Tsunami by Formula distills and simplifies the products from the Semi-Analytical Approach

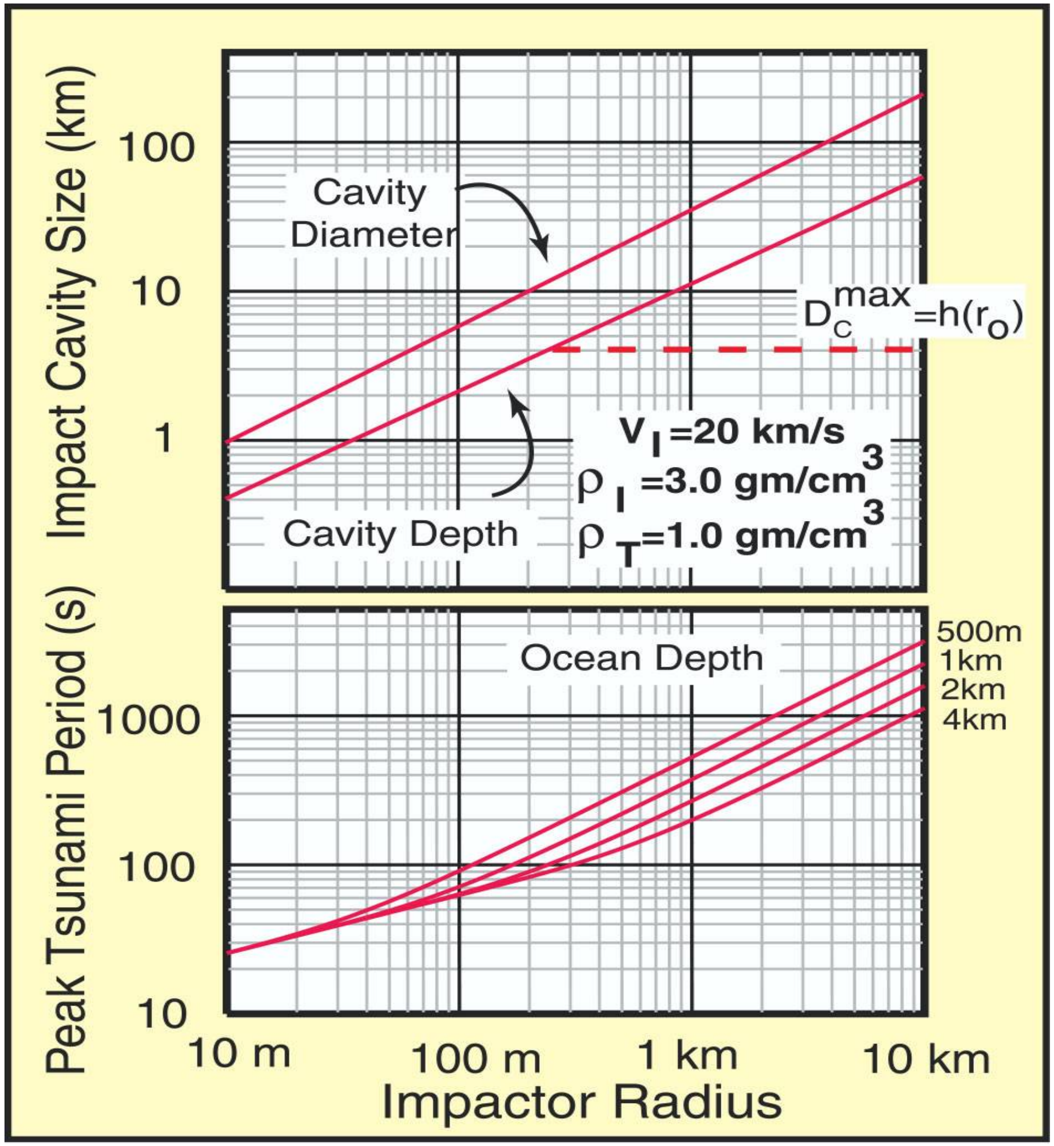
# Run Up Flow Chart



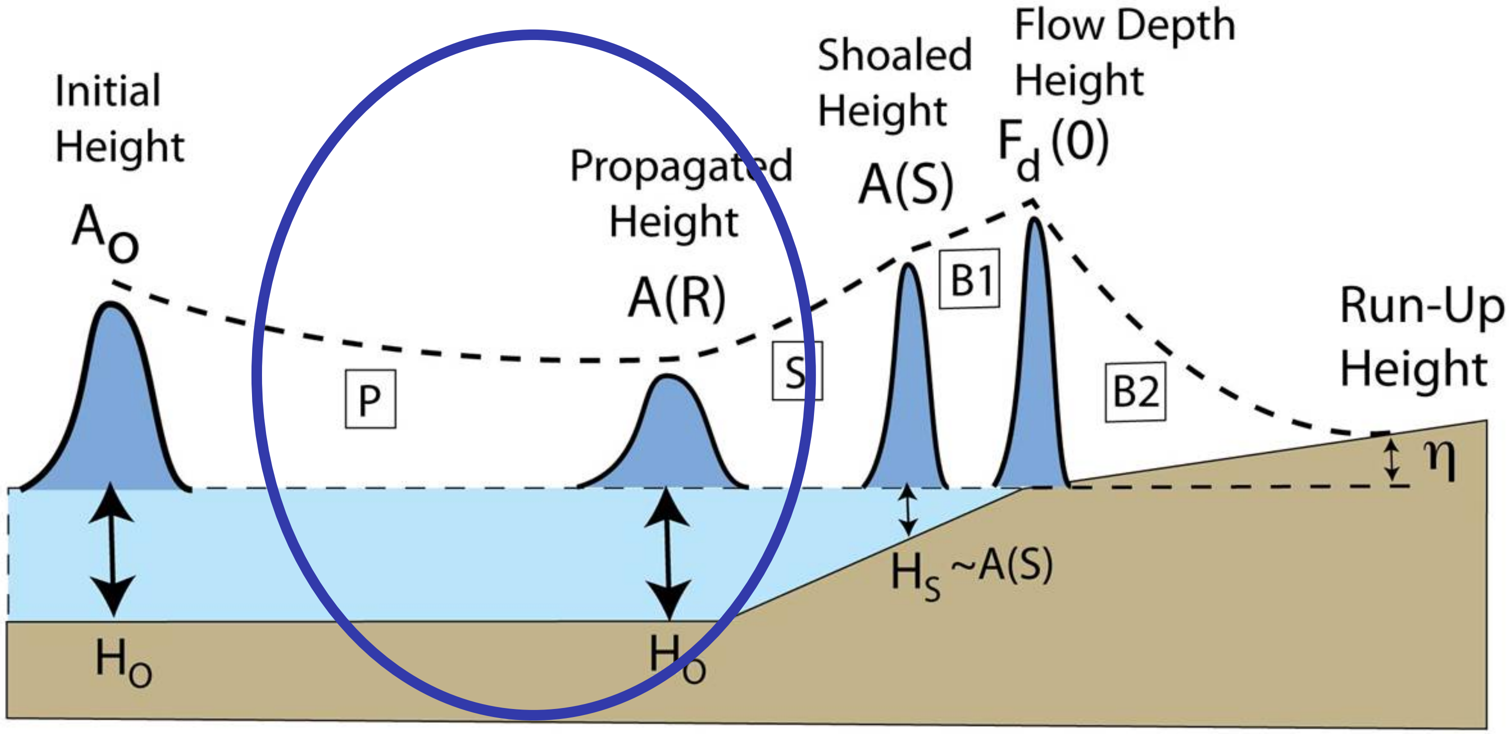
# Step 1. Initial Tsunami Height A0 and Diameter D

Use typical cavity depth-diameter relations again.

Asteroid cavities are radially symmetric so initial tsunami height (depth) A0 and diameter D are fixed regardless of observer direction.



# *Run Up Flow Chart*




Scalar $\boxed{P}$ Takes $A_o$ To $A(R)$

Scalar $\boxed{S}$ Takes $A(R)$ To $A(S)$

Function $\boxed{B1}$ Takes $A(S)$ To $F_d(0)$

Function $\boxed{B2}$ Takes $F_d(0)$ To $\eta$

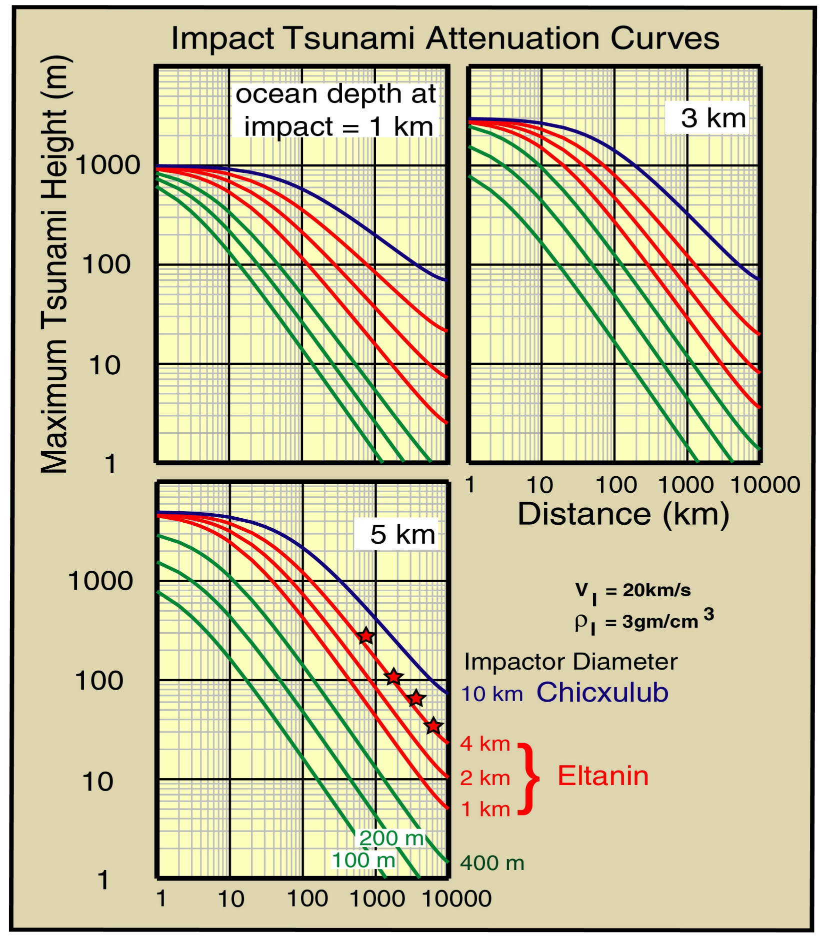
# Step 2. Propagation From Source to Shallower Water

Fit many curves with R being the distance from the source, D being the diameter of the cavity, and $H_0$ being the water depth at the source:

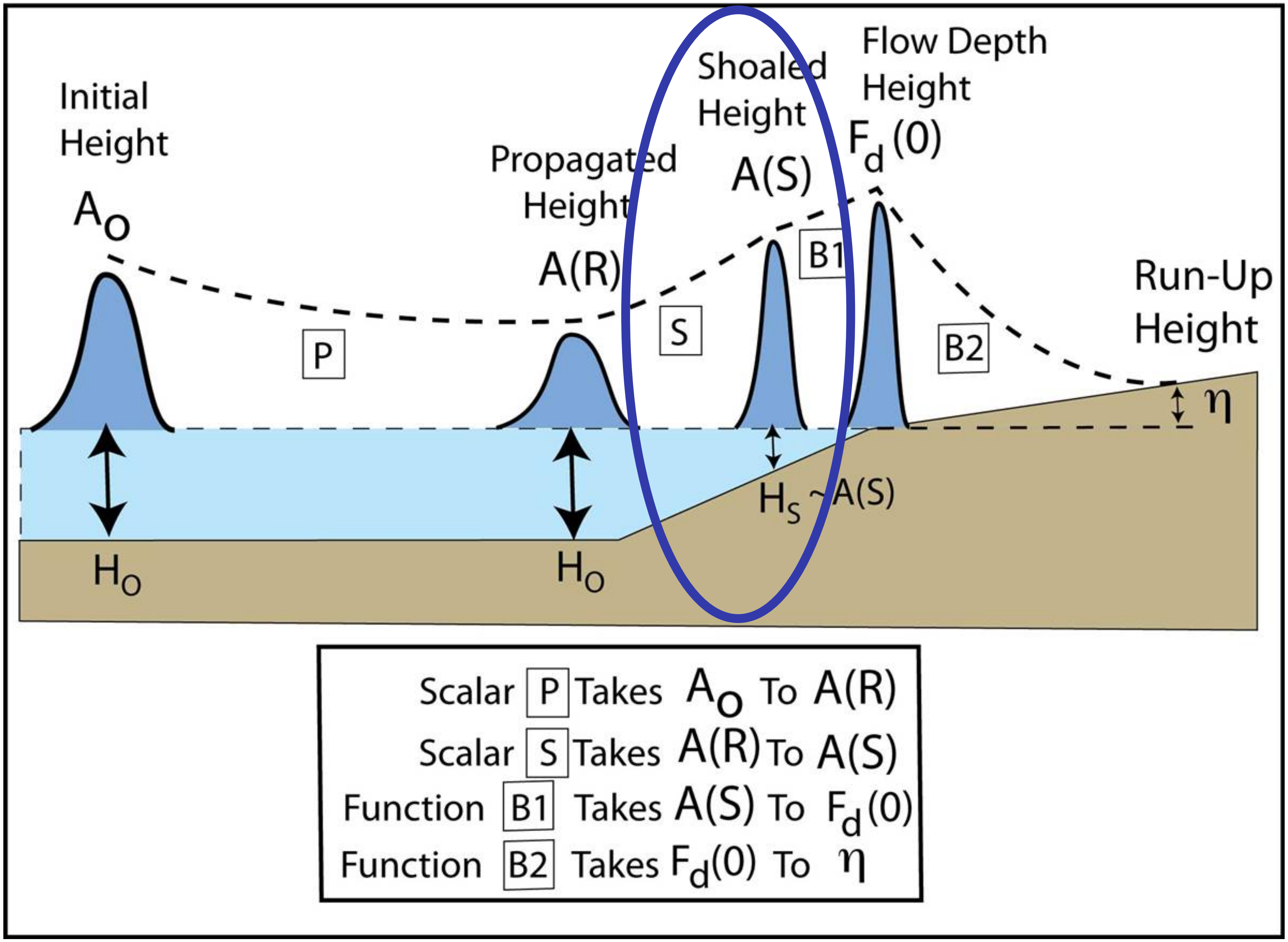
$$P = \left(1 + \frac{2R}{D}\right)^{-\left[0.5+0.575\exp\left(-0.0175\frac{D}{H_0}\right)\right]}$$

P is independent of $A_0$ but dependent on the ratio of $D/H_0$. Smaller sources (i.e., lower $D/H_0$) yield lower P because of DISPERSION is stronger for smaller events.

- $P < 1$, so wave size at distance is less than the initial amplitude $A(R)<A_0$.
- The first term in accounts for geometrical spreading. The second term in accounts for additional wave height losses due to frequency dispersion.



# *Run Up Flow Chart*



# Step 3. Tsunami Shoaling

- Tsunami waves slow and GROW as they move into shallower water because their wave energy gets compressed vertically and horizontally. This effect is called SHOALING

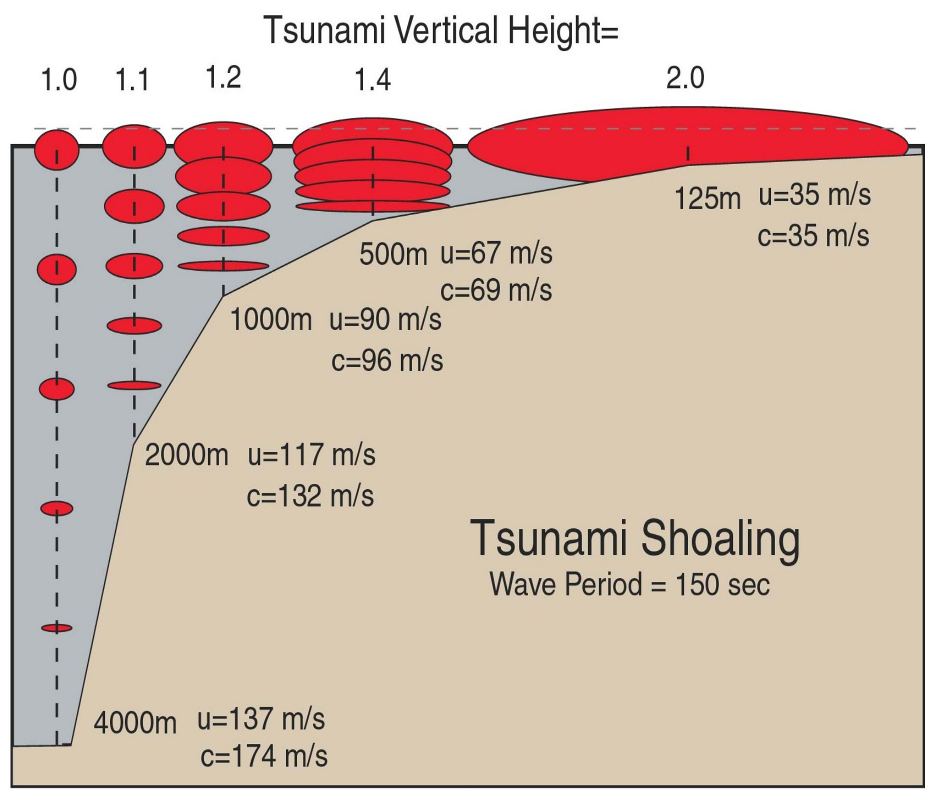
Shoaling factor "S" > 1 and takes A(R) to A(S) by

$$A(S) = A(R)\,S = A(R)\left(\frac{H_0}{H_s}\right)^{1/4}$$

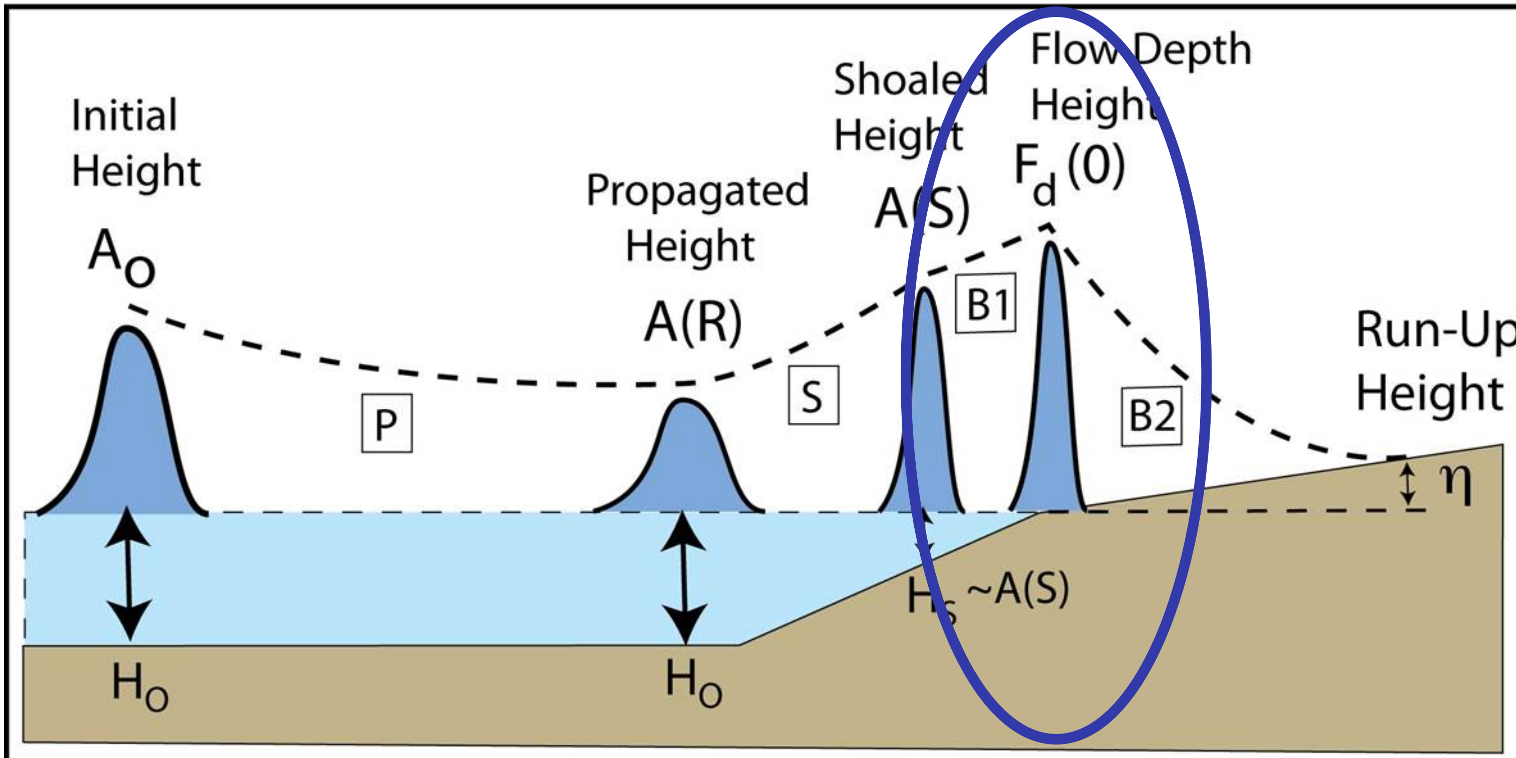
S is conservative because:

(1) it is the largest correction applicable to long waves. Shorter dispersive waves would actually grow less.

(2) No additional frictional losses are included in moving across the continental shelf into shallow water.



# Run Up Flow Chart




Scalar P Takes $A_o$ To A(R)
Scalar S Takes A(R) To A(S)
Function B1 Takes A(S) To $F_d(0)$
Function B2 Takes $F_d(0)$ To η

# Step 4. Tsunami Beaching

- Tsunami can't get bigger and bigger forever as they move into shallower and shallower water. Eventually they reach a terminal size in a process called BEACHING.
- Beaching is a complex, non-linear process that depends on beach slope, wave size, and period; however, it can be generalized in a fairly simple way.
- Beaching function "B1" takes shoaled height A(S) to $F_d(0)$, the flow depth at the beach.

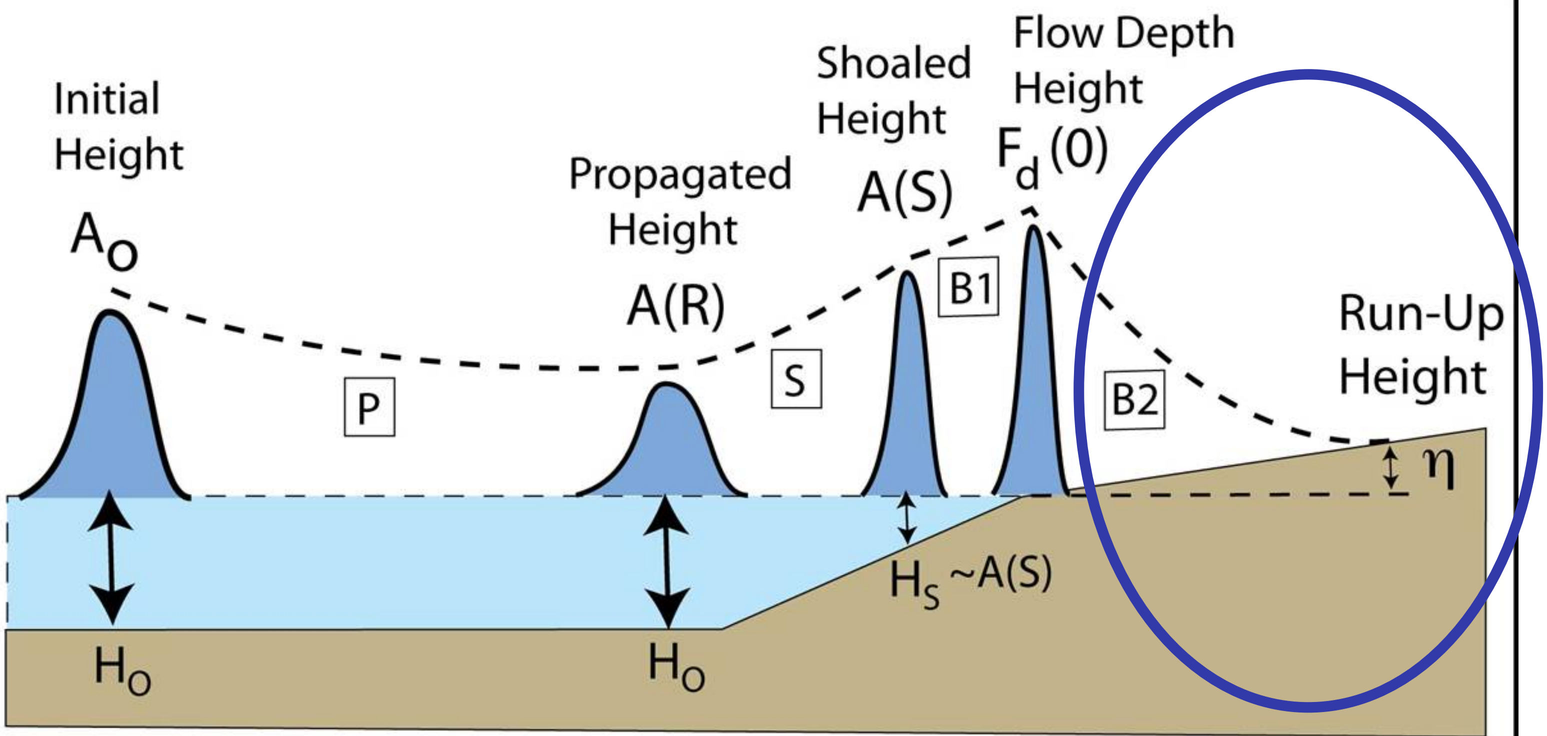
$$F_d(0) = h = A(S)^{4/5} H_S^{1/5}$$

Plug A(S) = A(R)S into above equation to yield:

$$F_d(0) = h = A(R)^{4/5} H_0^{1/5}$$

$F_d(X_0)$ is flow depth at the beginning of the run-in/run-up computation.

# Run Up Flow Chart




Scalar $\boxed{P}$ Takes $A_o$ To $A(R)$
Scalar $\boxed{S}$ Takes $A(R)$ To $A(S)$
Function $\boxed{B1}$ Takes $A(S)$ To $F_d(0)$
Function $\boxed{B2}$ Takes $F_d(0)$ To $\eta$

# Step 5. Tsunami Run-up

When the wave starts to run over dry land, friction and topography act to make it smaller with distance inland

$$\frac{dF_d(X)}{dX} = -\left[\frac{16.7n^2}{F_d(0)^{0.33}} + \frac{dT(X)}{dX}\right]$$

– $T(X)$ is the topographic elevation in meters.

– $F_d(X)$ is the flow depth at inland position X.

Integrate this equation until $F_d(X)$ vanishes.

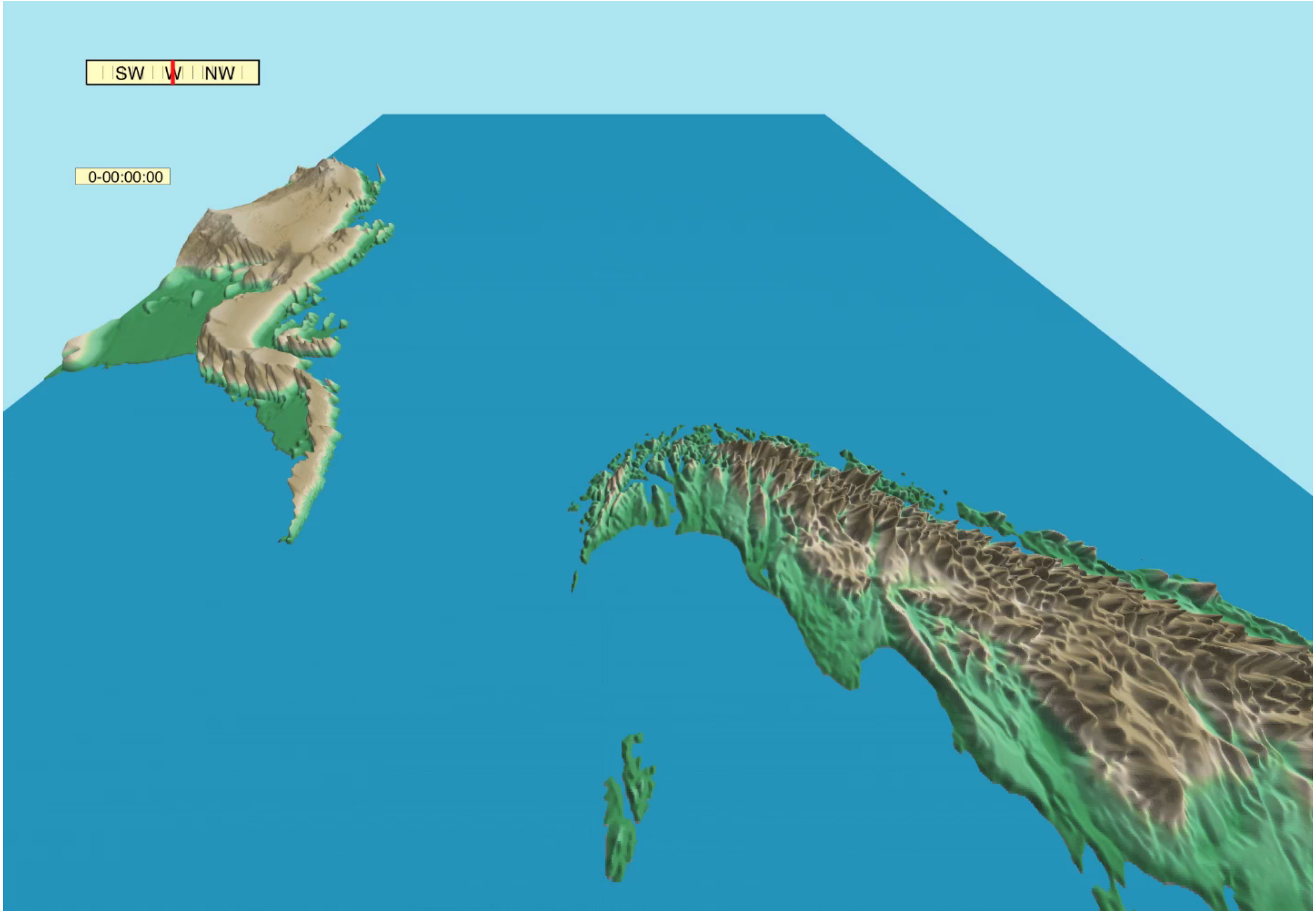
The resulting $X_{max}$ is the run-in distance and $T(X_{max})$ is the run-up height.

# Tsunami by Formula

*Pros:* Can't get much simpler, basically an EXCEL spreadsheet. Can be used by anyone with no prior experience. Resulting formula can be integrated over distance, impactor size, and time to get long term "hazard" estimates. Very few inputs needed.

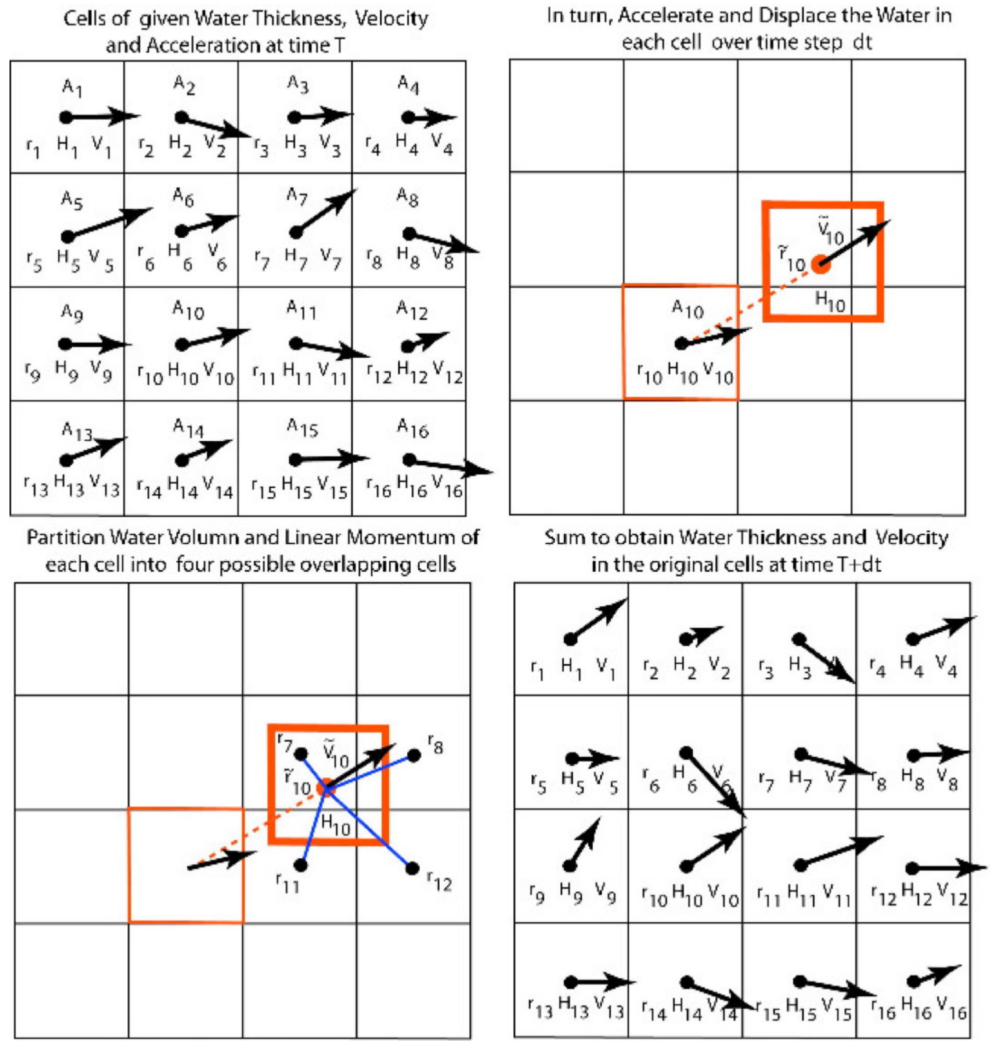
*Cons:* Only one or two numbers comes out – run up/ run-in. Arguably it produces worst case, "clear view" results. Error estimates are foggy. Largely based on linear theory. User has to depend on "Steve Ward" parameterization.

# Tsunami Squares



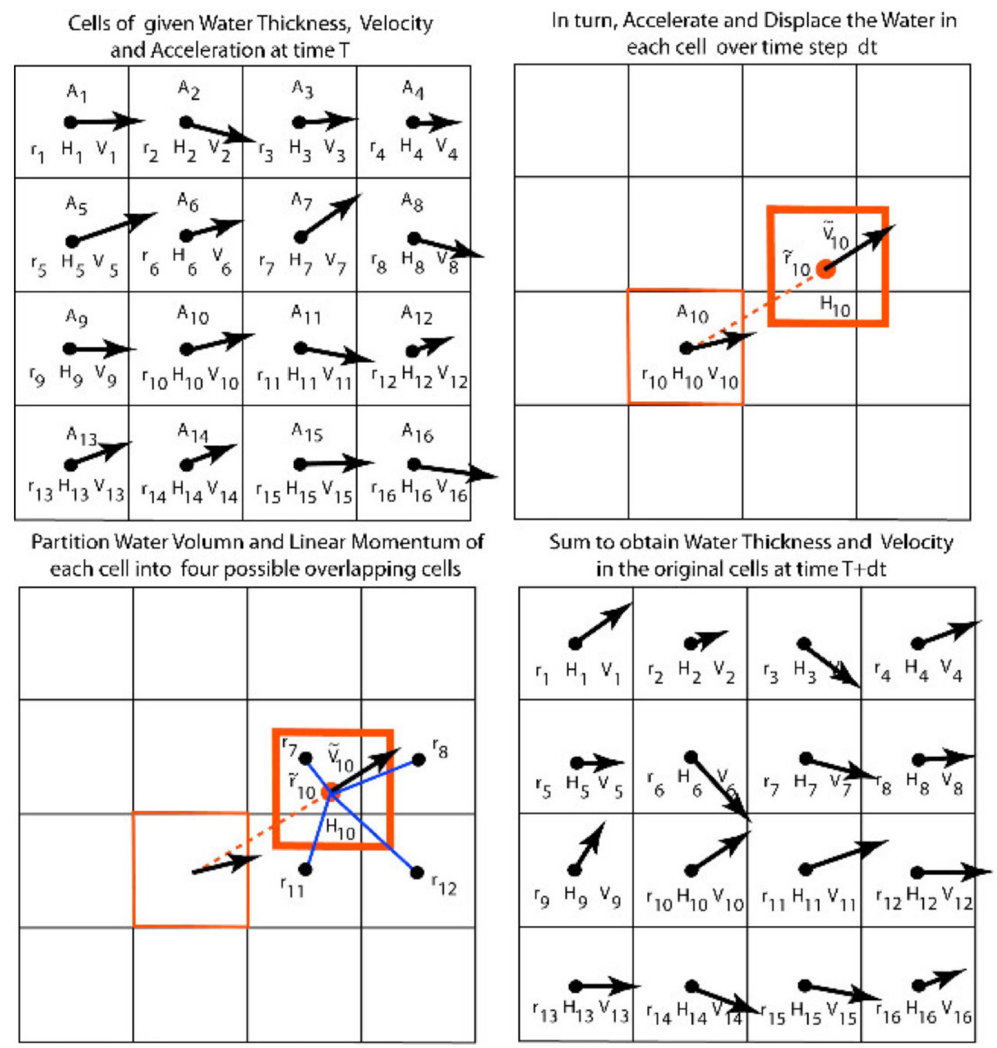
Tsunami Squares is a intuitive, versatile and straightforward means to handle a wide variety of flow-type problems fluid or “solid”.

(1) Given a fixed square grid of cells with known fluid thickness, average velocity and average acceleration



(2) In turn, accelerate and displace each square to a new position reached after a time interval of dt.

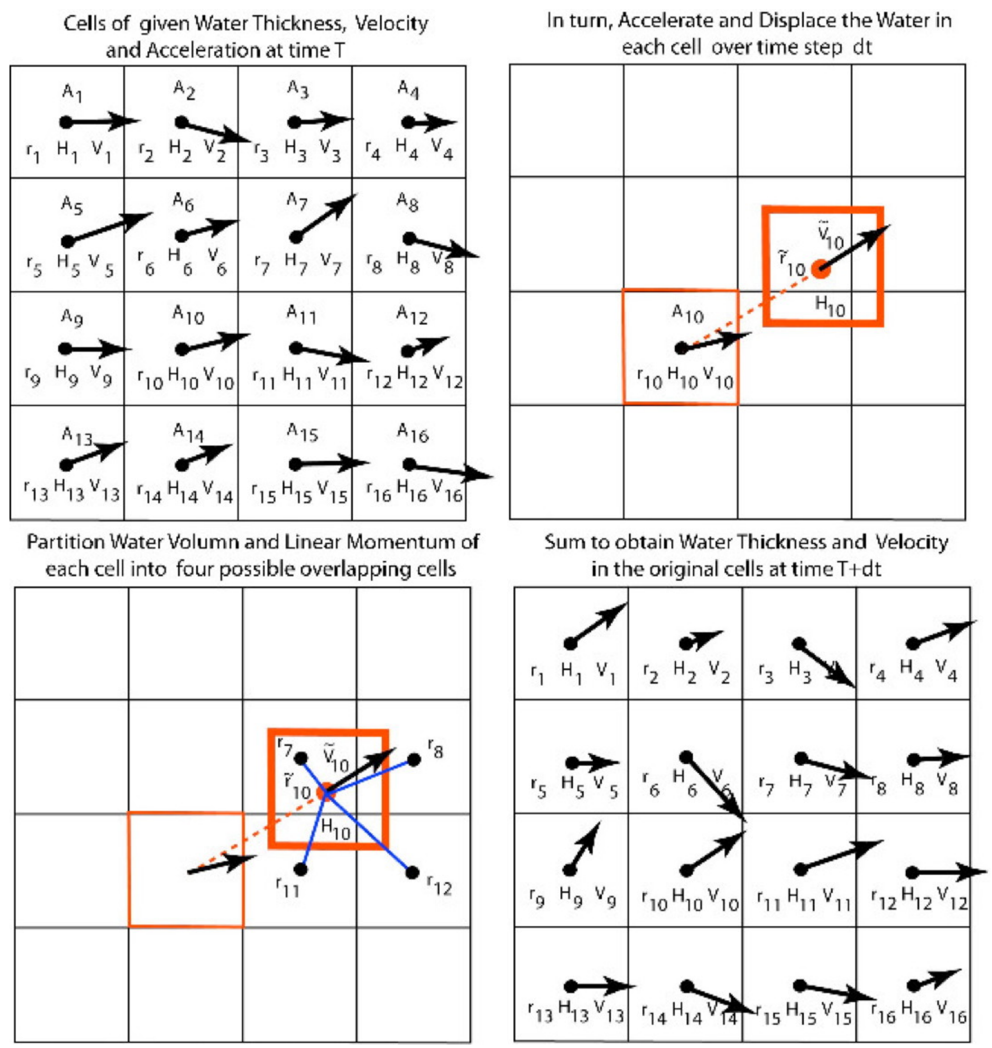
(3) Partition the mass and linear momentum of the displaced square in the four possible overlapping cells of the fixed grid.



(4) Sum the contributions of all displaced cells to get the thickness and mean velocity of all the squares on the fixed grid at time t+dt.

(5) Compute a new mean acceleration of the squares based on the slope of the "surface" and other forces/ frictions.

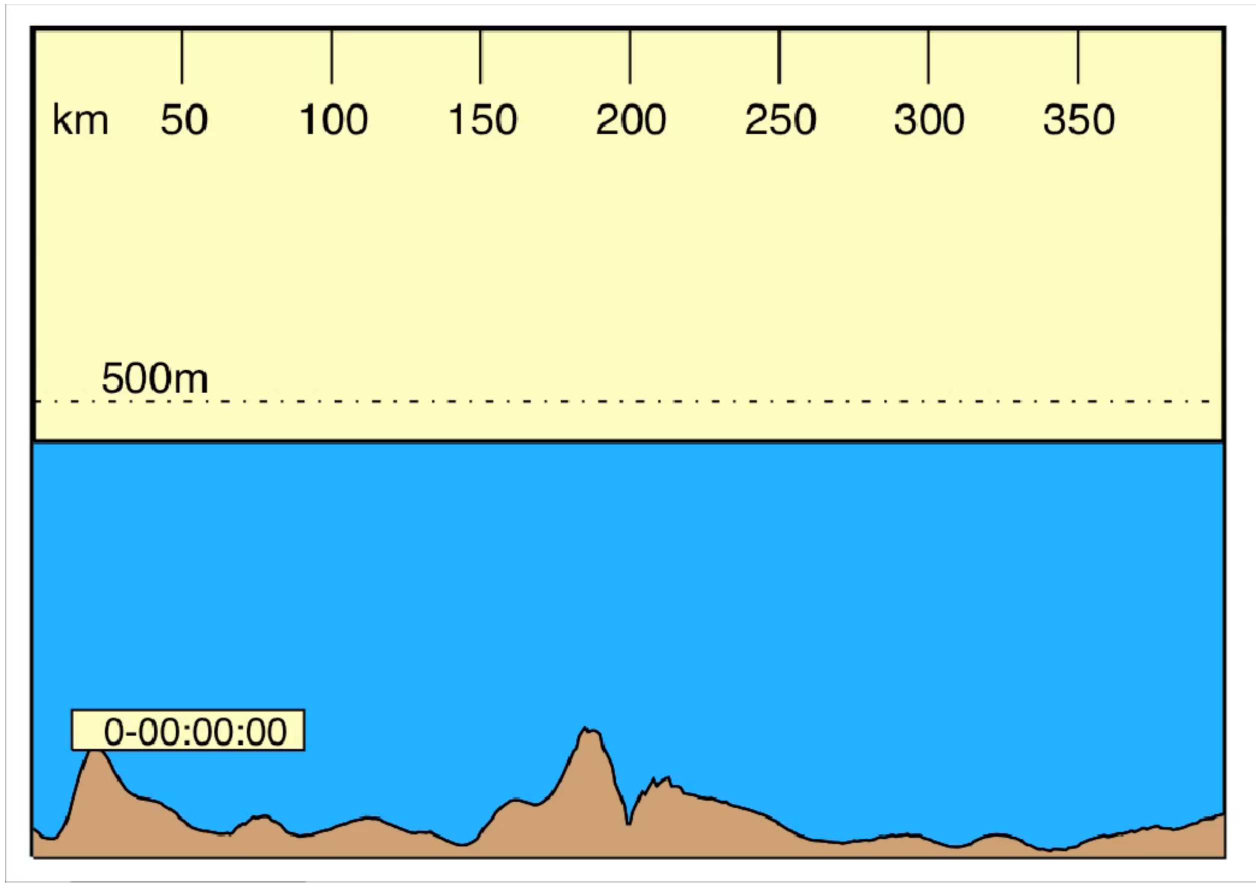
(6) Repeat.



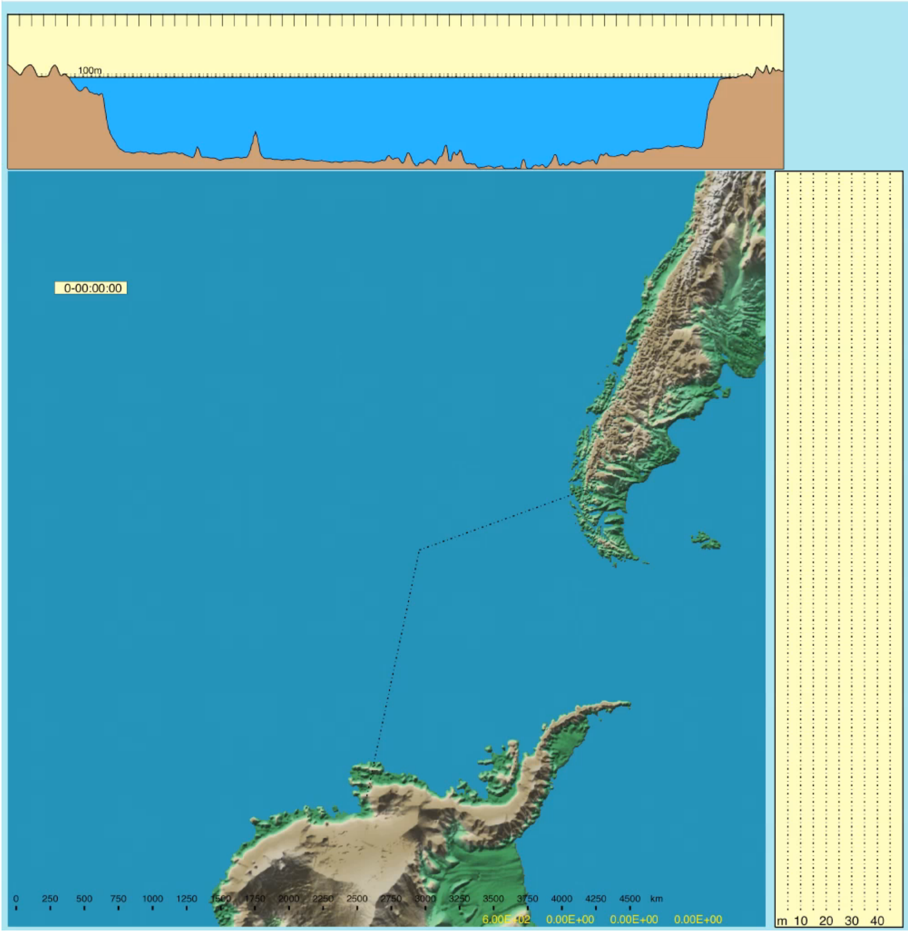
As applied to impacts, Tsunami Squares needs the initial shape of the transient cavity and the mean horizontal velocity of the ocean volume. Much like the semi-analytical approach, I get these from scaling relations and application of “geophysical license”.

*A Squares application to impact:*

*Non-linear now, but reminiscent of the Semi-Analytical result before.*



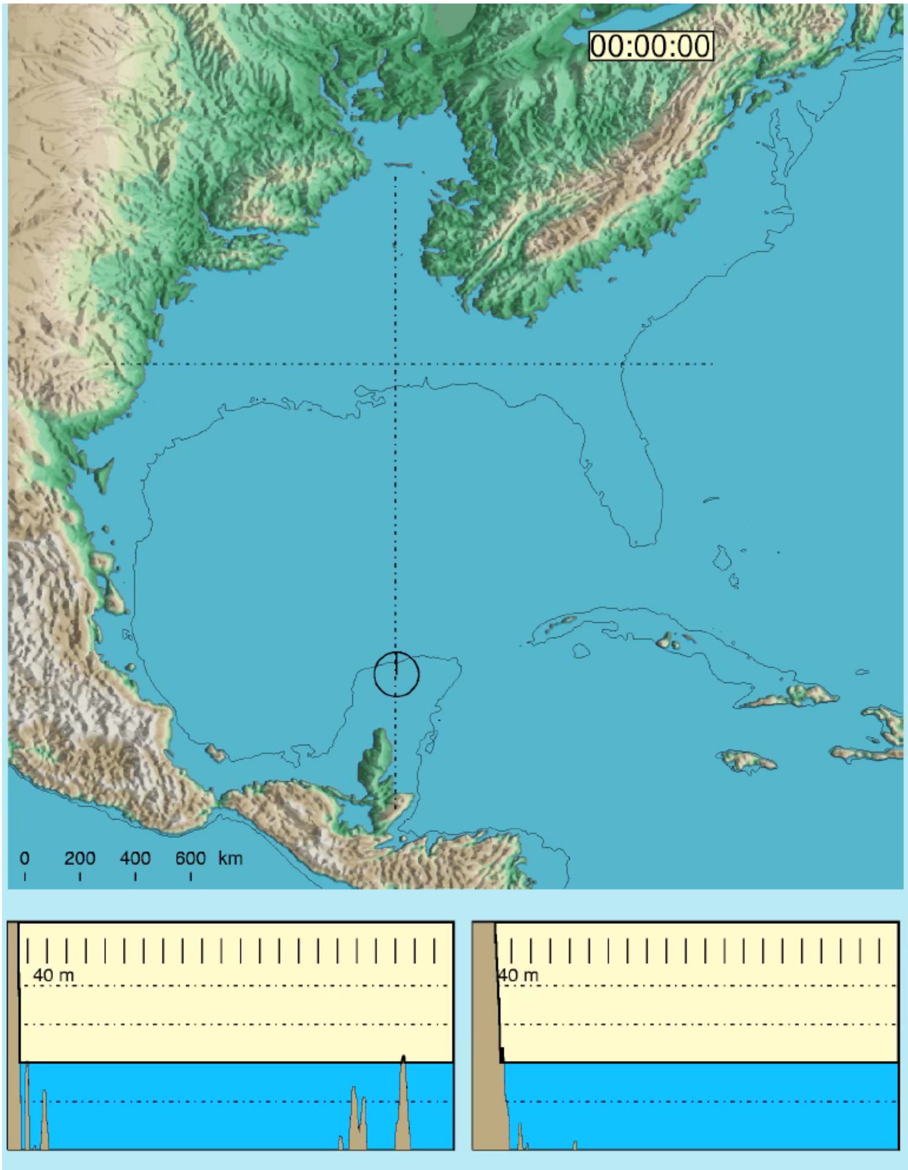
# Eltanin (again)
# 2Ma
# South Pacific

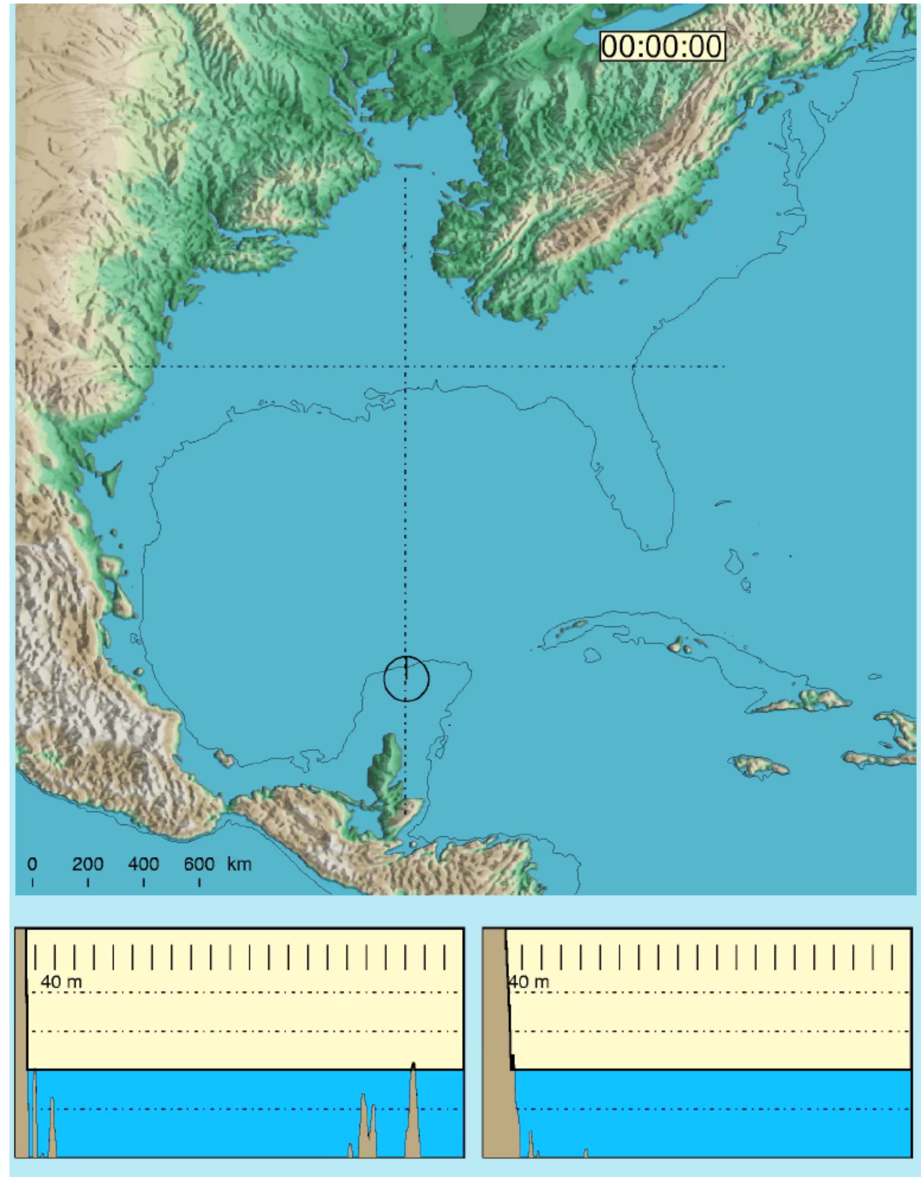




*50m waves at Antarctica.*
*30m waves at Chile*

# Chicxulub Sims.
# Normal and Oblique Impacts



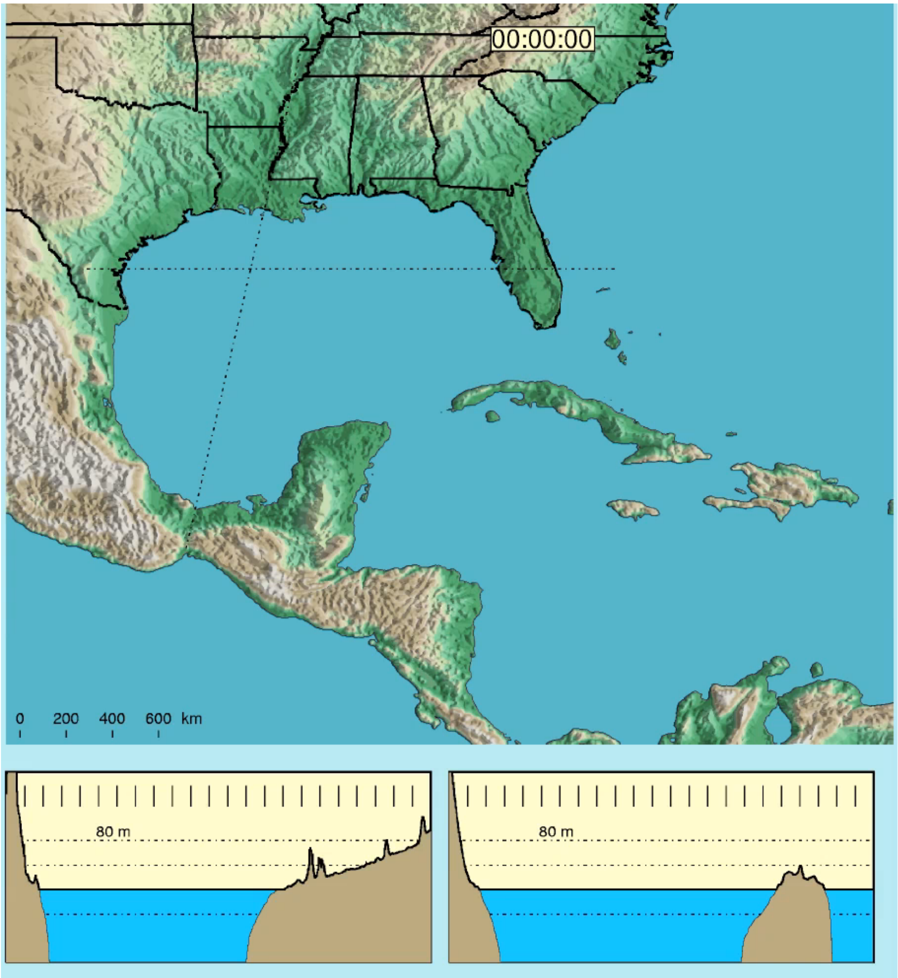



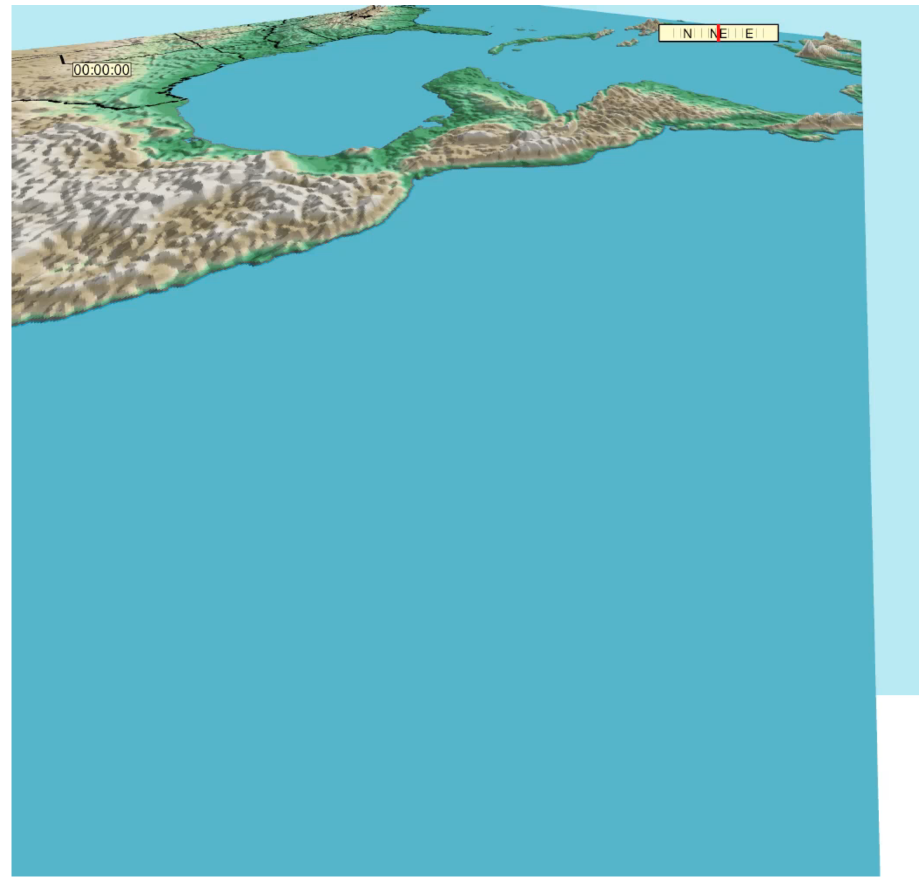
# Tsunami Squares

*Pros:* Fully nonlinear. No equations to solve. Can carry waves to shore or onto land. Includes all reflections and multi-paths. Makes beautiful movies!

*Cons:* Purely numerical approach. Can be time consuming depending on the number of squares and duration of signal. There's always a concern about numerical stability and numerical attenuation. Uses depth-averaged assumptions, but dispersion can be included. Results are complex and possibly hard to interpret physically.

# Thanks for Listening




*Goodbye Florida*